Modeling Optimal Investment and Greenhouse Gas Abatement in the Presence of Technology Spillovers

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Abstract : Game theory is frequently used to discuss climate change actions as it is an effective tool for modeling strategic behaviours. However, many existing extensive-form climate games often use discrete decision nodes for actions such as abatement, which is too simplistic. Furthermore, many models do not factor in technological developments that could potentially impact abatement costs in the future. In this thesis, I introduce a two-period, two-country game with continuous choices of abatement levels and investment spending. I examine this model under different technological scenarios, using backward induction to solve for the optimal levels of abatement and investment in abatement technologies. I find that investment levels are higher in the presence of specific technology spillovers, which results in greater welfare. How investment spending is distributed between the two countries is dependent on whether technology spillovers exist and whether countries are duplicating their investment efforts. The technology spillover effect is greater when countries are specialized and investing in different abatement technologies. Finally, though a single-decision-maker, i.e. an international government organization, will always produce a Pareto superior solution, the strategic interaction model in the presence of technology spillovers is the most accurate description of real world climate discussions.

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1 Introduction

In the last few decades, scientists and world leaders have collectively agreed that anthropogenic greenhouse gas emissions are the cause of climate change. However, there are a large range of views regarding how we should address the climate issue. Mitigation, adaptation, and geoengineering are three distinct responses to tackling climate change (Gordijn and ten Have, 2012). Mitigation is defined as actions taken to reduce emissions of greenhouse gases, for example switching from fossil fuels to renewable energy. On the other hand, adaptation involves measures taken to minimize the negative impacts of climate change, such building levees in coastal regions. Lastly, geoengineering refers to large-scale manipulation of Earth's climate system, such as spraying reflective aerosols into the atmosphere, to reduce the effects of climate change. In addition to many ethical concerns, geoengineering involves many technologies that are still in the process of being developed (Gordijn and ten Have, 2012). There are currently too many uncertainties related to geoengineering, which makes it unsuitable for analysis within a game theoretic model.

Part of this thesis explores how countries decide on optimal levels of carbon emissions abatement. Climate change mitigation and emissions abatement involve the same actions, but "abatement" is more clearly defined as the reduction of emissions from Business-as-Usual (BAU) levels. I chose mitigation rather than adaptation as the focus of this thesis for a multitude of reasons: 1) Mitigation requires bearing costs now, with benefits only felt by later generations. In contrast, adaptation benefits are mostly felt by the generation bearing the costs. 2) Mitigation involves local costs, but global benefits while adaptation involves local costs and local benefits. 3) Mitigation has a narrow focus on carbon emissions, while adaptation can be very broad and heterogeneous (Brown, 2010). The temporal aspect of the first two factors suggests that strategic interactions of mitigation are more interesting to study and the third factor implies that mitigation is an easier action to model.

Even within the scope of mitigation, there are debates about how much abatement should be achieved and the time frame. One extreme side tends to argue for immediate cuts in global emissions despite short-terms costs to GDP as well as the political and technical infeasibility (Jewell and Cherp, 2020). Other groups are more optimistic about potential technological breakthroughs in the future and tend to argue for less emissions abatement in the short run (Li and Wang, 2017). Between these conflicting views, the United Nations Environment Programme (2019) has declared that we must reduce global cumulative emissions by 15 gigatonnes before 2030 in order to keep global temperature increase below 2ºC.

Despite the urgency to reduce emissions, it is incredibly difficult to create and ratify international climate agreements, let alone reach compliance (Shishlov et al., 2016). One main reason is that international government organizations have little power to enforce climate treaties. Another underlying reason is that our atmosphere is a prime example of a public good. A country's private marginal cost of polluting a gigaton of carbon dioxide is less than the social marginal cost experienced by the globe. This results in the overproduction of carbon emissions. Furthermore, climate change mitigation involves cooperation between multiple countries. Since welfare depends on collective action, each country's commitment to abatement levels will depend on what other countries are expected to do.

Both emissions abatement and technological investment can create positive externalities. Abatement from one country decreases emissions of the entire globe, which benefits neighboring countries as well. Investment in research and development (R&D) can also create technology spillovers if knowledge or capital are shared unintentionally, allowing those who didn't share the costs of investment to also benefit. The game theoretic model in this thesis suggests that investment levels are higher in the presence of technology spillovers, resulting in greater welfare levels. Furthermore, the impact of technology spillovers greatly depends on whether countries are duplicating investment efforts or not. The positive effect of technology spillovers is much greater for countries that are investing in different technologies as it results in more diverse abatement methods and lower abatement costs in the future.

2 Background

Climate change negotiations have been modeled extensively using game theory. Among the existing literature, climate change game theoretic models tend to fall into two categories: two-player games versus large coalition-style games (Hsu, 2010; Wood, 2010). The function of these models can generally be sorted into intellectual exercises versus applicable models intended to facilitate real-world negotiations (Madani, 2013). Many these models use arbitrary parameters to define their model and consist of decision nodes that are discrete. For example, in the two-period model from Hsu (2010), both players can choose to abate or not to abate in both time periods. When boiling down most of these models into a static form game, they are essentially the Prisoner's Dilemma (shown below).

		Country X	
		Reduce emissions	Don't reduce emissions
Country Y	Reduce emissions	3,3	1,4
	Don't reduce emissions	4,1	2,2

Table 1: The typical Prisoner's Dilemma used to describe climate change mitigation between two countries.

The Prisoner's dilemma can be a useful game for understanding the simple reasons causing the breakdown of cooperation. When looking at the payoffs for Country X and Country Y, you can see that both countries choosing to reduce emissions (top-left corner) would lead to a better payoff than both choosing to not reduce emissions (bottom-right corner). However, each country will always defect and choose "Don't reduce emissions" since that is the dominant strategy, resulting in a Pareto inferior outcome. While this game does represent some of the underlying issues of climate change cooperation, it is over-simplistic. In real life, countries don't choose between abating emissions and not abating at all. Existing models serve their purpose in informing us that collective abatement is the optimal solution, however few models tackle what level of optimal abatement we should aim for. Should it be exactly 15 gigatonnes of CO_2 by 2030 as the IPCC suggests? Or is the current marginal cost of abatement too high for us to reach that goal?

Another gap in existing literature involves the potential impact of abatement technologies. While we know there are links between investment in R&D, innovation, and technological advancements that reduce abatement costs, these relationships aren't fully defined yet (Heal and Tarui, 2010; Baker et al., 2008). Despite our lack of knowledge, the potential of enhanced abatement technology cannot be dismissed as it could greatly decrease future costs of emissions reduction. Hence, investment in abatement technology plays a key role in my game theoretic model.

There are currently models such as Integrated Assessment Models (IAMs) that draw connections between human development, societal choices and the science of climate change. IAMs such as the DICE model currently only factor in exogenous technological change, but modelling investment in R&D as an endogenous factor is starting to gain more popularity (Barron, 2013). One challenge of treating investment as an endogenous variable is that technological innovation has many forms, some of which may pivot or shift the marginal abatement cost (MAC) curve down or even increase it (Amir et al., 2008). Baker et al. (2006) looked at three different R&D programs and found that optimal R&D was very ambiguous and was heavily dependent on the parameters. Due to lack of empirical evidence, I used the most optimistic assumption presented in (Amir et al., 2008) that a given level of investment would lead to a pivot-down of the MAC curve.

3 The Model

3.1 Introducing the Baseline Models

The strategic model I propose is a two-period, two-player model consisting of continuous decision nodes. Each player, a country, chooses their abatement level and investment spending in period 1, and abatement level in period 2. The countries, labelled as X and Y, make their decisions simultaneously in period 1 (2020-2030) and period 2 (2030-2040). Each period in this model takes place over the course of ten years. This was done because climate action and technological improvements do not happen instantly. It takes years for investment in R&D to pay off as improvements in technology. Furthermore, the 2019 IPCC report broadly states that cumulative abatement of 15 gigatonnes of carbon is needed before 2030 to keep global temperature increase below 2°C (United Nations Environment Programme, 2019). The report does not have a strict guideline about how much abatement needs to be achieved per year, so using a ten-year period was the best option.

Initially, the welfare functions were intended to be grounded in empirical data. However, while conducting my literature review I came across challenges in finding observational data to support relationships between carbon emissions abatement and GDP, or relationships between investment in technology and its effect on technological improvement. In review papers that compiled cost and benefit functions from different Integrated Assessment Models, the magnitude of these functions would vary significantly between different models (Diaz and Moore, 2017). Furthermore, I had the added complication that unless I had very specific parameters, the models did not have valid best response functions to solve for the solution, let alone a global maximum. I ended up using the `Manipulate` command in Mathematica to toggle each parameter, including the discount factor, to ensure that the functions actually could be optimized. In the end, due to all of these factors, I chose to use welfare as the output since it has an ordinal value. This way, the magnitude of parameters should not affect the analysis as I make my comparisons.

3.1.a The Baseline Strategic Model

The baseline strategic model has two welfare functions, one for each country. Since each country is acting in their own best interest, Country X is optimizes W_x while Country Y is optimizes W_y. The payoffs in period 2 are discounted by a discount factor of 0.9. For both countries, period 2 can be considered a sub-game of period 1, as the optimal level of abatement is dependent the investment spending in period 1. Lastly, W_x and W_y have the same parameters, so Country X and Country Y have symmetrical welfare functions.

$$W_x = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1x}}^2 - i_x + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1 + \frac{i_x}{2})}] \tag{1}$$

$$W_y = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1y}}^2 - i_y + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2y}}^2}{2(1 + \frac{i_y}{2})}] \tag{2}$$

Where:

a_{1x} = Abatement of Country X in period 1

a_{1y} = Abatement of Country Y in period 1

a_{2x} = Abatement of Country X in period 2

a_{2y} = Abatement of Country Y in period 2

i_x = Investment of Country X in period 1

i_y = Investment of Country Y in period 1

3.1.b The Baseline Optimal Model

Strategic interactions often lead to sub-optimal outcomes, especially when positive externalities are involved. Abatement produces positive externalities because when one country reduces emissions, it reduces the global emissions as well, which benefits everyone.

In order to compare these sub-optimal outcomes to the optimal outcome, I created a single welfare function, W_T, that is a sum of W_x and W_y. By having a single-decision-maker optimize W_T, I am maximizing W_x and W_y without their self-interests competing against each other. If X and Y were firms, this would be the equivalent of the government intervening to make decisions for both players. However, at the country level, international government agreements can rarely be enforced, therefore this is just labelled as the single-decision-maker model, or the optimal model.

$$\begin{aligned} W_T = 40\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}({a_{1x}}^2 + {a_{1y}}^2) - (i_x + i_y) \\ + 0.9[40\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1 + \frac{i_x}{2})} - \frac{{a_{2y}}^2}{2(1 + \frac{i_y}{2})}] \end{aligned} \tag{3}$$

3.2 Explaining Each Component of the Model

3.2.a The Benefit of Abatement in Period 1 (and Period 2)

$20\ln(a_{1x} + a_{1y} - 14)$ is the term capturing the benefit of collective abatement in period 1. As mentioned earlier, countries need to achieve cumulative abatement of 15 gigatonnes of carbon emissions between now and 2030. If Country X and

Country Y reach 15 gigatonnes of carbon abatement, this term is zero, essentially reflecting a somewhat stable outcome in 2030. 15 gigatonnes is essentially the “carbon threshold”. If collective abatement is less than 15 gigatonnes, there will be climate damages, hence a negative benefit. However, if collective abatement is greater than 15 gigatonnes, there will a positive benefit due to avoided climate damages.

Current IAMs predict that with increasing temperatures, climate damages will increase exponentially (Morris et al., 2012). Since global temperature increase is a function of pollution; higher pollution levels lead to increased global temperatures, which lead to greater climate damages (see Fig. 1a). If abatement is defined as a decrease from Business-as-Usual (BAU) emissions, then abatement levels below the “carbon threshold” will lead to a greater temperature increase, which results in more climate damages (see Fig. 1b). The natural logarithmic function reflects this exponential increase in marginal damages. As abatement shrinks (moving from right to left), marginal benefit becomes more negative at an increasing rate since the slope becomes steeper. However, there is one disadvantage of using the natural logarithmic function. Collective abatement levels less than or equal to 14 will cause the benefit term to be undefined, creating a lower limit for abatement levels in the model.

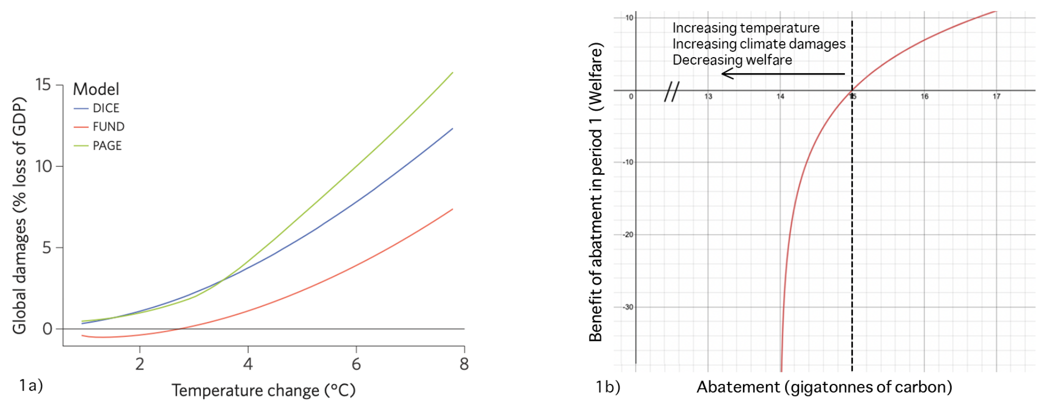

Figure 1: 1a) Multiple IAMS show an exponential relationship between climate damages and temperature change (Morris et al., 2012). 1b) As abatement decreases below the “carbon threshold” of 15 gigatonnes, benefit of abatement (welfare) becomes more negative at an increasing rate.

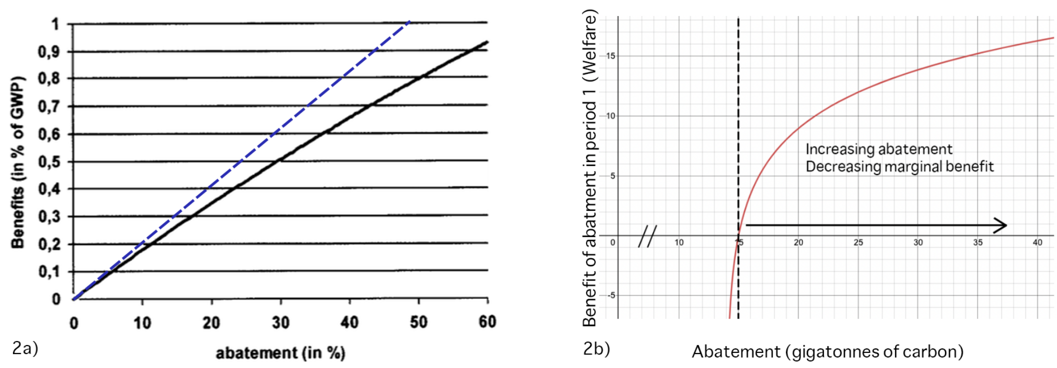

Figure 2: 2a) The estimated global benefit of emissions reduction in percentage of GWP (Hamaide and Boland, 2000). The dashed blue line was added by the author to emphasize the concavity of the benefit function. 2b) The natural logarithmic function reflects decreasing marginal benefits for abatement levels above the ”carbon threshold” of 15 gigatonnes.

The other reason why a natural logarithmic function was chosen as the abatement benefit curve is because its shape above the x-axis reflects decreasing marginal benefits of abatement. Cost and benefit projections from Hamaide and Boland (2000) show that as percent abatement increases, benefits as percentage of global world product will increase at a decreasing rate (see Fig. 2a). This matches up with the shape of the natural logarithmic function quite well (see Fig. 2b). One important note is that Hamaide and Boland used measures of percent abatement, meaning the percentage reduction from BAU emissions. In contrast, my model uses gigatonnes of carbon as the unit of abatement. The translation from percent abatement to gigatonnes of abatement should not be problematic, since my model uses ordinal rather than cardinal values as its output. Therefore, the shape of the function matters more than the actual magnitude of the function.

Similarly, the $20\ln(a_{2x} + a_{2y} - 14)$ term represents the benefit of abatement in period 2. Empirical data about how much carbon emissions we need to abate between 2030 to 2040 is not as clearcut. I made an assumption that we should follow the trajectory of 15 (cumulative) gigatonnes of abatement over 10 years. Therefore, I decided to keep the same "carbon threshold" of 15 gigatonnes in period 2. This term, along with the period 2 total cost of abatement term, $-\frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}$, is multiplied by 0.9, since these costs and benefits are felt in period 2.

3.2.b The Total Cost of Abatement in Period 1

$-\frac{1}{2}{a_{1x}}^2$ is the term capturing the total abatement cost (TAC) of Country X in period 1. By taking the derivative of this term with respect to a_{1x}, we get $-a_{1x}$, the marginal abatement cost (MAC) term. Marginal cost of abatement increases (in terms of absolute value) because we use cheaper mitigation methods to abate the first few units of CO_2 and switch to more expensive technologies for the later units of CO_2. The polynomial TAC term in my model is line with functions from various IAMs and emissions predictions models (Hamaide and Boland, 2000; Morris et al., 2012; Ackerman and Bueno, 2011). Besides the direct cost of abatement, abating emissions also has indirect costs to economic development. Abatement may involve cutting back on production, thereby lowering GDP. I decided not to include this effect in the welfare function as it was not worth the added complexity, however, I thought this point should still be addressed.

3.2.c The Cost of Investment

$-i_x$ is the term reflecting the cost of Country X's investment in period 1. The model assumes that R&D industry has constant-returns to scale. That is, the investment spending is proportional to the amount of technological innovation that is produced. Though investment spending itself is a cardinal value with units of billions of dollars, the $-i_x$ term within the welfare function becomes an ordinal value.

3.2.d The "Cost Modifier"

$\frac{1}{1+\frac{i_x}{2}}$ is the "cost modifier" term and captures the impact of investment from period 1 on abatement costs in period 2. If investment is zero, then no technological innovation happens, so the cost modifier is equal to one, and the TAC expression in period 2 is the same as period 1. The cost modifier decreases asymptotically with i_x, showing that technological innovation (and investment spending) does decrease abatement costs, but at decreasing-returns-to-scale. (Kim, 2018).

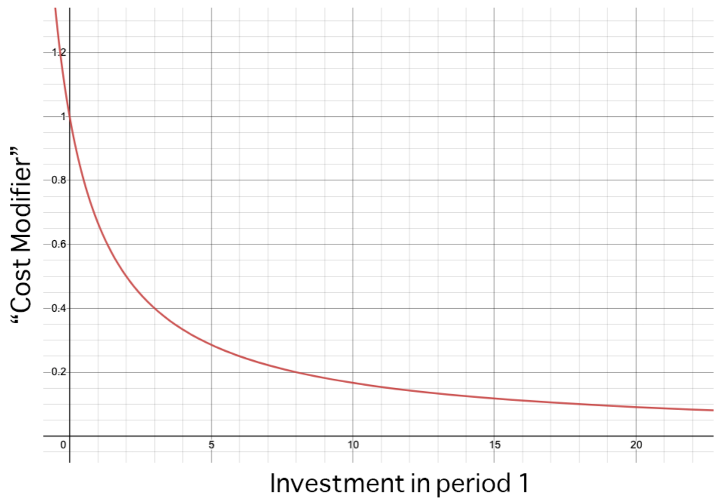

Figure 3: The function for the "cost modifier" highlights the decreasing-returns-to-scale of innovation on period 2 total abatement costs. Since technological innovation and investment are proportional, an asymptotic relationship also exists between the "cost modifier" and investment spending.

3.2.e The Total Cost of Abatement in Period 2

$-\frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}$ is the term capturing the TAC of Country X in period 2. It takes the TAC expression from period 1, $-\frac{1}{2}{a_{1x}}^2$, and multiplies it with the "cost modifier" term $\frac{1}{1+\frac{i_x}{2}}$. This impact can be described as a pivot down of the TAC curve. Past studies of technological innovation's effect on marginal abatement cost curves show varying impacts such as a pivot down of the MAC curve, a shift down, or a combination of a shift and a pivot (Amir et al., 2008). Since there was little empirical evidence for which impact was most realistic, I went with the pivot since it was the easiest to model.

4 Solving the Baseline Model under Different Scenarios

As I've mentioned above, actions such as emissions abatement, or any other action that produces positive externalities usually lead to sub-optimal outcomes. Technology spillover is another example of a positive externality. In private industry, technology spillovers can increase the productivity of other firms, without them having to share the costs of the research and development (Bloom et al., 2013). Technology spillovers in the context of my model can explain how one country's investment in R&D can also lower total abatement costs for the other country. I am interested in how strategic and optimal outcomes change in the presence and absence of technology spillovers. The baseline models will be examined under the following circumstances (each scenario will be explained in more detail):

1. There are no technology spillovers.
2. There are technology spillovers; both countries invest in similar abatement technologies.
3. There are no technology spillovers, but technology can be bought.
4. There are technology spillovers; both countries invest in different abatement technologies.

4.1 Scenario 1: There are No Technology Spillovers

This scenario operates under the assumption that both countries are able to keep their technological developments secret, creating zero spillovers. In period 1, both countries decide on abatement level and investment level. The benefits of abatement are shared, however the benefits of investment are not. In period 2, Country X's optimal choice of abatement level will solely depend on how much investment they made in period 1, and vice versa for Country Y. Their welfare functions are exactly the same as the baseline strategic model, and this is a symmetric game.

4.1.a Solving the Strategic Model

$$W_x = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1x}}^2 - i_x + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}] \qquad (1, \text{revisited})$$

$$W_y = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1y}}^2 - i_y + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2y}}^2}{2(1+\frac{i_y}{2})}] \qquad (2, \text{revisited})$$

Since this is a two-period game, I use backward induction to solve for each country's optimal solution. Backward induction is useful as it identifies the optimal action in the second period, then uses this information to determine the optimal action in the first period.

Each country has three decision variables, and each variable has a continuous decision node. This requires creating three Best Response (BR) functions for Country X and three BR functions for Country Y. Best response functions map out the strategy which produces the best outcome for a player when taking other player's strategies as a given (Fudenberg and Tirole, 1991). Note: the following is quick explanation of the backward induction solving process; detailed steps and calculations are available in Appendix A.

Starting the process of backward induction, I differentiate W_x with respect to a_{2x} and differentiate W_y with respect to a_{2y}. I then find the best response functions of a_{2x} and a_{2y}:

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} + \sqrt{276 - 28a_{2y} + {a_{2y}}^2 + 40i_x}\right) \qquad (4)$$

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} + \sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}\right) \qquad (5)$$

As seen from Equations (4) and (5), a country's best response function for abatement in period 2 is a function of their own investment as well as the period 2 abatement level of the other country. The intersection of both countries' best response functions give me the strategy profile at which each country is playing a best response to the other. I substitute (4) and (5) into each other to get the intersection. This creates a strategy profile (a^*_{2x}, a^*_{2y}) that is on both countries' best responses functions.

$$a^*_{2x} = \frac{14 + 7i_x + \sqrt{(2+i_x)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y} \qquad (6)$$

$$a_{2y}^* = \frac{14 + 7i_y + \sqrt{(2+i_y)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y} \tag{7}$$

Now that I have the strategy profiles for a_{2x}^* and a_{2y}^* as Equations (6) and (7), I substitute them back into the original welfare functions, leaving us with W_x and W_y in terms of a_{1x}, a_{1y}, i_x and i_y. Since abatement and investment are chosen at the same time in period 1, it shouldn't matter which decision variable I choose to solve for next. I then follow the same process above for a_{1x} and a_{1y}, finding their best response functions:

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}\left(14 - a_{1y} + \sqrt{276 - 28a_{1y} + {a_{1y}}^2}\right) \tag{8}$$

$$BR_{a_{1y}}(a_{1x}) = \frac{1}{2}\left(14 - a_{1x} + \sqrt{276 - 28a_{1x} + {a_{1x}}^2}\right) \tag{9}$$

As seen from Equations (8) and (9), each country's best response function for abatement in period 1 is a function of the abatement level of the other country. In the real world, we should expect some sort of trade-off between abatement levels and investment spending due to a finite budget. However, I did not factor this interaction into the model in order to avoid too much complexity. Now that I have the BR functions for a_{1x} and a_{1y}, I substitute (8) and (9) into each other to find the strategy profile (a_{1x}^*, a_{1y}^*) where both countries' best responses functions intersect. This gives us the values:

$$a_{1x}^* = \frac{1}{2}(7 + \sqrt{89}) \tag{10}$$

$$a_{1y}^* = \frac{1}{2}(7 + \sqrt{89}) \tag{11}$$

Now that I have the strategy profile (a_{1x}^*, a_{1y}^*), I substitute it back into the original welfare functions. At this point, the welfare functions of Country X and Country Y are expressed only in terms of i_x and i_y. I work through the same process of differentiating W_x with respect to i_x, and differentiating W_y with respect to i_y. I won't include the best response functions for i_x and i_y since they are ridiculously long (they will be included in Appendix A). At this point, the best response functions were too complex to be plugged into each other and solved. I worked around this problem by plotting levels curves where $\frac{\partial W_x}{\partial i_x}$ and $\frac{\partial W_y}{\partial i_y}$ equal zero (see Figure 4). These level curves graphed out the best response functions for i_x and i_y, and I found the intersection point by using the `Apply@RegionIntersection` command in Mathematica.

With that, I've found the strategy profile (i_x^*, i_y^*), which is (1.67, 1.67). I substitute these numbers back into Equations (6) and (7) to find a_{2x}^* and a_{2y}^*. Now that I have solved for all the decision variables, I can substitute them back into W_x and W_y. These are the solutions of the strategic model under a scenario with no technology spillovers:

$$i_x = i_y = 1.67$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{1x} = a_{1y} = 9.03$$
$$W_x = W_y = -12.39$$

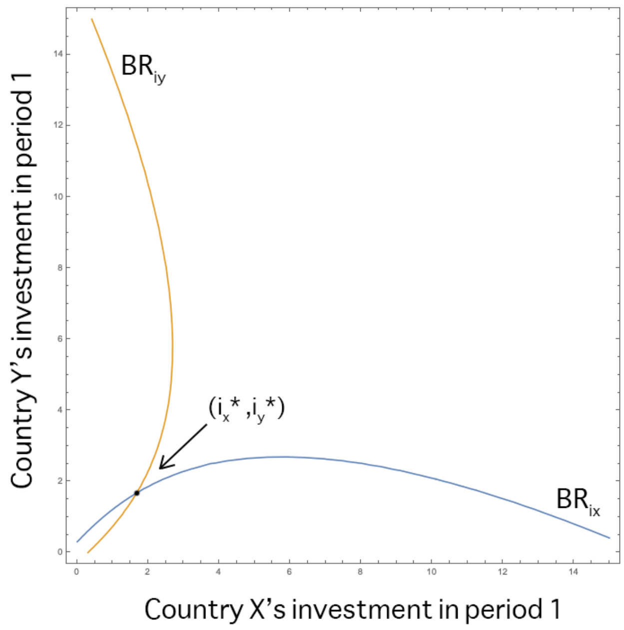

Figure 4: Graphing the BR functions for i_x and i_y to find the strategy profile (i_x^*, i_y^*).

4.1.b Solving the Optimal Model

In the single-decision-maker model, each country's TAC in period 2 is still a function of their individual investment. The total welfare function below, W_T, is simply the sum of W_x and W_y, and is the same as the optimal baseline model introduced earlier. By making this a one-player model, strategic interactions are removed, though the single-decision-maker still has six variables to solve for. Since there is only one player acting, this is a decision theory model.

$$W_T = 40\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}({a_{1x}}^2 + {a_{1y}}^2) - (i_x + i_y) + 0.9[40\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})} - \frac{{a_{2y}}^2}{2(1+\frac{i_y}{2})}] \quad \text{(3, revisited)}$$

Backward induction is also used to solve this model (detailed steps and calculations are in Appendix B). First, I find the optimal values of a_{2x} and a_{2y} by differentiating W_T with respect to a_{2x}, and then differentiating W_T with respect to a_{2y}. Similar to before, I find the best response functions of a_{2x} and a_{2y}.

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} + \sqrt{356 - 28a_{2y} + {a_{2y}}^2 + 80i_x}\right) \quad (12)$$

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} + \sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}\right) \quad (13)$$

The BR functions for a_{2x} and a_{2y} in this model are different compared to the BR functions for a_{2x} and a_{2y} in the strategic model (see Equations (4) and (5)). This is because the period 2 benefit of abatement in the optimal model is $0.9[40\ln(a_{2x} + a_{2y} - 14)]$ whereas the benefit of abatement for an individual country in the strategic model is $0.9[20\ln(a_{2x} + a_{2y} - 14)]$. Therefore, we should expect the amount of abatement in period 2 to be greater in the optimal model compared to the strategic model. I substitute (12) and (13) into each other to find the optimal decision profile:

$$a_{2x}^* = \frac{14 + 7i_x + \sqrt{(2+i_x)^2(129 + 20i_x + 20i_y)}}{4 + i_x + i_y} \quad (14)$$

$$a_{2y}^* = \frac{14 + 7i_y + \sqrt{(2+i_y)^2(129 + 20i_x + 20i_y)}}{4 + i_x + i_y} \tag{15}$$

Now that I have the decision profiles for a_{2x}^* and a_{2y}^* as Equations (14) and (15), I substitute them back into W_T, so that W_T is only in terms of a_{1x}, a_{1y}, i_x and i_y. I then follow the same process above for a_{1x} and a_{1y}, finding their best response functions:

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}\left(14 - a_{1y} + \sqrt{356 - 28a_{1y} + {a_{1y}}^2}\right) \tag{16}$$

$$BR_{a_{1y}}(a_{1x}) = \frac{1}{2}\left(14 - a_{1x} + \sqrt{356 - 28a_{1x} + {a_{1x}}^2}\right) \tag{17}$$

Now that I have the BR functions for a_{1x} and a_{1y}, I substitute (16) and (17) into each other to find the decision profile, (a_{1x}^*, a_{1y}^*), where both countries' best responses functions intersect. This gives us the values:

$$a_{1x}^* = \frac{1}{2}(7 + \sqrt{129}) \tag{18}$$

$$a_{1y}^* = \frac{1}{2}(7 + \sqrt{129}) \tag{19}$$

Now that I have the decision profile (a_{1x}^*, a_{1y}^*) as (18) and (19), I substitute it back into the total welfare function. W_T is now expressed only in terms of i_x and i_y. I do the same process of differentiating W_T with respect to i_x, then differentiating W_T with respect to i_y. Once I find the BR functions for i_x and i_y (which were also too long to put in this section), I use the same technique of plotting the level curves out to find the intersection of the best response functions.

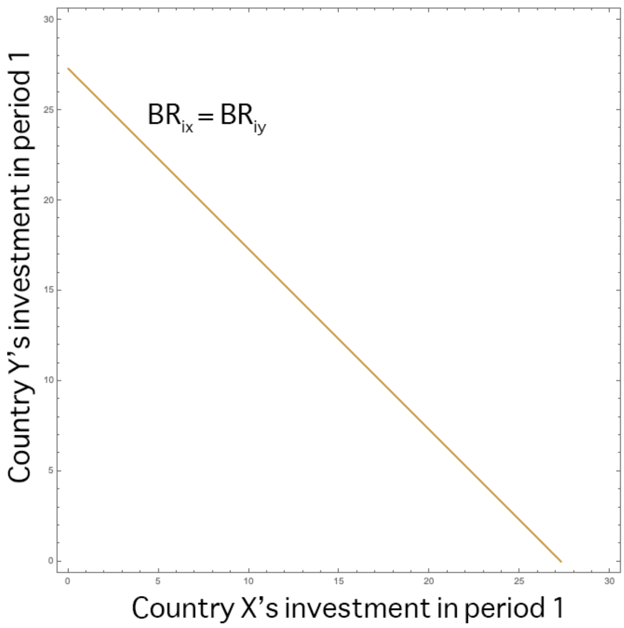

Figure 5: Graphing the BR functions for i_x and i_y.

In Figure 5, there appears to be just one BR function. However, there are two BR functions in the graph, they are just perfectly overlapping. Any point along this curve is a solution that maximizes W_T. This means there are an infinite number of combinations of i_x and i_y that maximize the total welfare function. However, if the single-decision-maker wants the decision profile that divides welfare evenly between both countries, then we need to apply the condition $(i_x^* = i_y^*)$. After making this condition, we get the solution (13.64, 13.64) for (i_x^*, i_y^*). I can substitute these numbers back into Equations (14) and (15) to find a_{2x}^* and a_{2y}^*. Now that I have solved for all the decision variables, I can substitute them back into W_T. These are the solutions of the optimal model under a scenario with no technology spillovers:

$$i_x = i_y = 13.64$$
$$a_{1x} = a_{1y} = 9.18$$
$$a_{1x} = a_{1y} = 16.49$$
$$W_T = 44.03$$

4.1.c Comparing the Strategic and Optimal Models

Scenario 1: Solutions

	Strategic	Government
i_x=i_y	1.67	13.64
a_{1x}=a_{1y}	8.22	9.18
a_{2x}=a_{2y}	9.03	16.49
Welfare (individual)	X: -12.39 Y: -12.39	X: 11.00 Y: 11.00
Welfare (total)	-24.77	22.01

Table 2: The strategic and optimal solutions for Scenario 1: No technology spillovers. Note: There will be slight discrepancies between the sum of the individual welfare and total welfare due to rounding error; this applies to all the other solutions as well.

First of all, in both the strategic and optimal models the solutions are symmetric, meaning: $a_{1x} = a_{1y}$, $a_{2x} = a_{2y}$ and $i_x = i_y$. This is because country X and country Y have identical parameters in their welfare functions. The optimal solution is Pareto superior compared to the strategic solution as it leads to greater total welfare as well as greater individual welfare for both countries. We can see that in the absence of technology spillovers, the optimal solution involves greater investment and higher abatement levels in both periods. One striking observation is how much greater abatement is for the optimal solution compared to the strategic solution, especially in period 2. This means either marginal cost of investment decreases in the optimal model, or marginal benefit of investment increases in the optimal model.

Country X's investment spending term, $-i_x$, is the same in W_T and W_x. Therefore, the marginal cost of investment should be the same in both strategic and optimal models. This suggests that there is a greater marginal benefit of investment in the optimal model that is driving this difference in investment levels between the two models.

The marginal benefit of investment can be divided into two forms: direct and indirect. The marginal direct benefit can be described as the decrease in total abatement costs in period 2. The marginal indirect benefit is the higher welfare caused by greater abatement levels in period 2, which are a result of the decreases of TAC in period 2. The term capturing the direct benefit of Country X's investment in the strategic model and optimal model is the same (see the last term in Equation (1) and the second-to-last term in Equation (3)). Since this term, $0.9[-\frac{{a_{2x}}^2}{2(1+\frac{i_x}{2}))}]$ is the same for both models, we can conclude that the direct marginal benefits of of i_x, are also the same for W_T and W_x.

This leaves the indirect marginal benefits of i_x as the source of different investment levels between the strategic and optimal solutions. In the strategic model, the term capturing the indirect benefit of Country X's investment is $0.9[20\ln(a_{2x} + a_{2y} - 14)]$. In the optimal model, the term capturing the indirect benefit of Country X's investment is $0.9[40\ln(a_{2x} + a_{2y} - 14)]$. In order to find the marginal indirect benefit of investment, I first rewrite these terms as functions of i_x and i_y . I substitute the strategy profile (a_{2x}^*, a_{2y}^*) into $0.9[20\ln(a_{2x} + a_{2y} - 14)]$ to get:

Indirect benefit of i_x in $W_x =$

$$18\ln[\frac{-28 - 7i_x - 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}] \quad (20)$$

I substitute the decision profile (a_{2x}^*, a_{2y}^*) into $0.9[40\ln(a_{2x} + a_{2y} - 14)]$ to get:

Indirect benefit of i_x in $W_T =$

$$36\ln[\frac{-28 - 7i_x - 7i_y + \sqrt{(2+i_x)^2(129+20i_x+20i_y)} + \sqrt{(2+i_y)^2(129+20i_x+20i_y)}}{4+i_x+i_y}] \quad (21)$$

In order to find the marginal indirect benefit of i_x in the strategic and optimal model, I take the partial derivative of Equations (20) and (21) with respect to i_x. The marginal indirect benefit terms ended up being very long, so I will not include them here. However, just by looking at Equations (20) and (21) we can infer that the marginal indirect benefit of i_x will be greater in W_T. This explains why investment is greater in the optimal model compared to the strategic model.

4.2 Scenario 2: There are Technology Spillovers; Both Countries Invest in Similar Abatement Technologies

This scenario operates under the assumption that there will be technology spillovers between the two countries. Furthermore, both countries are investing in the same abatement technologies, e.g. both are only investing in renewable wind energy. Essentially, there is a duplication of investment efforts. The technology spillovers are modelled such that in period 2, both countries will have the "cost modifier" of the country that invested more in period 1.

There are four possibilities: 1) Country X invests more than Country Y; $i_x > i_y$, 2) Country Y invests more than Country X; $i_y > i_x$, 3) Both countries invest the same (positive) amount because they assume the other country is not investing, and 4) Neither country invests because they both assume the other country is investing.

I start off by assuming that Country X invests more than Country Y (possibility 1), so both countries will be using the "cost modifier" $\frac{1}{1+\frac{i_x}{2}}$. Thus, Country X's TAC in period 2 is $-\frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}$ and Country X's TAC in period 2 is $-\frac{{a_{2y}}^2}{2(1+\frac{i_x}{2})}$.

4.2.a Solving the Strategic Model

$$W_x = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1x}}^2 - i_x + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}] \quad (22)$$

$$W_y = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1y}}^2 - i_y + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2y}}^2}{2(1+\frac{i_x}{2})}] \quad (23)$$

The process of using backward induction to solve this strategic model is very similar to the process used to solve the strategic model from Scenario 1. Therefore, I won't walk through all the steps again, nor will I include detailed calculations for this problem in the Appendix. Instead, I will just note the differences in this model compared to the strategic model in Scenario 1.

First of all, the welfare functions W_x and W_y are no longer symmetric. Both countries' total abatement costs in period 2 are dependent on Country X's investment spending in period 1. Since Country Y's investment spending has no impact in period 2, Country Y will choose to have zero investment in order to maximize their welfare. Since $i_y = 0$, Country X doesn't have a best response function for i_x in terms of of i_y. Instead, Country X just maximizes W_x with respect to i_x to get their optimal investment level. These are the solutions of the strategic model under a scenario where there are technology spillovers, duplication of investment efforts, and $i_x > i_y$:

$$i_x = 13.84, i_y = 0$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{2x} = a_{2y} = 13.06$$
$$W_x = 5.41, W_y = 19.25$$

This may seem like the end of the backward induction process. However, the final solution for the strategic case hasn't been solved yet. If you recall, there are four possibilities in this strategic game: 1) $i_x > i_y$, 2) $i_y > i_x$, 3) $i_x = i_y$, where i_x and $i_y > 0$, and 4) $i_x = i_y = 0$. The Nash Equilibrium above was found assuming that $i_x > i_y$, so if we look at Figure 6, I've only traveled through the top branch of the game tree (the arrows with a red dash). I've only solved for the first possibility. In possibility 2: the NE would be the reverse, where Country Y invests more and the welfare outcomes are flipped.

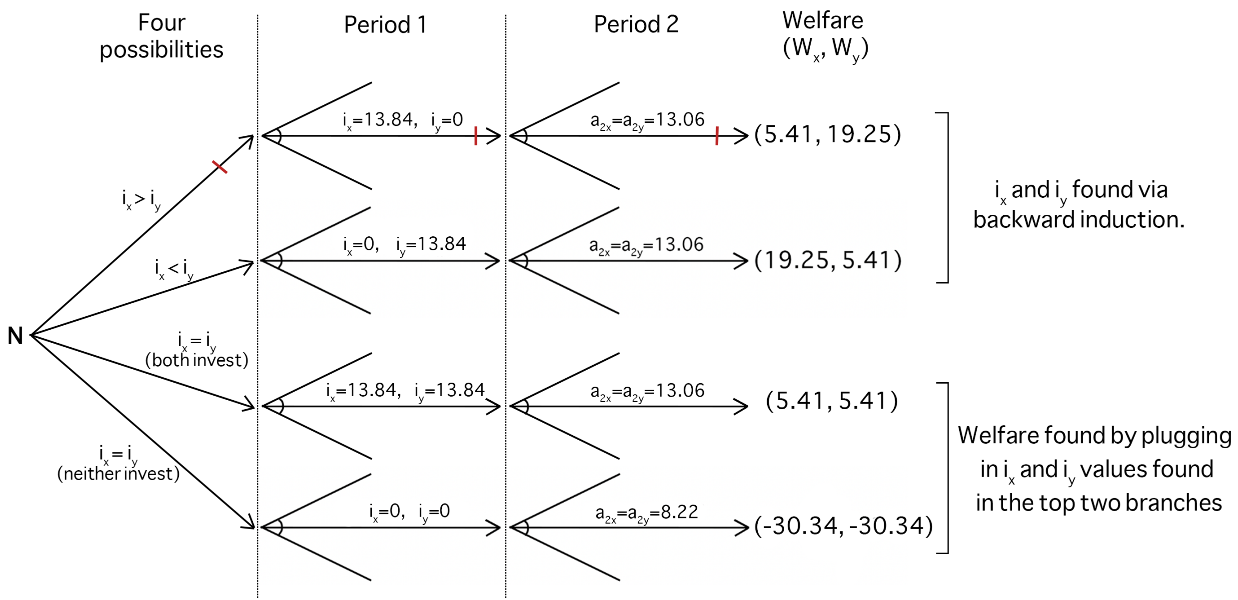

Figure 6: The game tree for the Scenario 2 strategic model. The beginning node, N, stands for Nature. Nature is random, so there is a chance that Country X and Country Y can travel along any one of the four branches. Though abatement level is a variable that both countries must choose in period 1, since it has no influence on the period 2 sub-game it was left out of the game tree.

Possibility 3: I entertain the possibility that both countries assume that the other won't invest, in which case the best response for each of them would be to invest 13.84. I substitute $i_x = i_y = 13.84$ back into their welfare functions to get (5.41 5.41) as their payoffs. Possibility 4: I must also entertain the possibility that both countries assume that the other country will invest, in which case the best response for each of them would be to not invest. I substitute $i_x = i_y = 0$ back into their welfare functions to get (-30.34, -30.34) as their payoffs. Now that we have the welfare outcomes for all four possibilities, we can write this model out as a static game (see Table 3).

			Country X	
			p	$1-p$
			$\mathrm{i}_x = 13.84$	$\mathrm{i}_x = 0$
Country Y	q	$\mathrm{i}_y = 13.84$	5.41, 5.41	5.41, 19.25
	$1-q$	$\mathrm{i}_y = 0$	19.25, 5.41	-30.34, -30.34

Table 3: Rewriting the Scenario 2 strategic model as a static game.

This 2×2 grid maps out the strategic interactions that happen on the leftmost side of Figure 5, before Country X and Y travel down one of the four branches. This static game is basically a version of "Chicken", a very well known anti-coordination game. In the context of my model, the "Chicken" is the country that "yields" by investing while the other country gets a higher payoff by "standing their ground" and not investing.

This game has three Nash Equilibria; the two pure NE are ($i_y = 13.84, i_x = 0$) and ($i_y = 0, i_x = 13.84$), also known as (invest, not invest) and (not invest, invest). The other NE is a mixed strategy Nash Equilibrium. Mixed strategies occur when the player uses a probability distribution to randomly choose among available choices in order to avoid being predictable. For example, in Table 3, Country X has a probability p of investing, and a probability of $1-p$ of not investing. Likewise, Country Y has a probability q of investing, and a probability of $1-q$ of not investing. To find Country X's best response to Country Y investing with probability q, I have to find Country X's expected payoffs from investing and not investing. Country X's expected welfare from investing is $E[W_x(\text{invest})] = q(5.41) + (1-q)(5.41) = 5.41$. Country X's expected welfare from not investing is $E[W_x(\text{not invest})] = q(19.25) + (1-q)(-30.34) = -30.34 + 49.62q$. Investing is a best reply to q when the payoff to investing is at least as good as the payoff to not investing. Therefore, to find the value of q that makes Country X choose to invest, we need to solve:

$$E[W_x(\text{invest})] \geq E[W_x(\text{not invest})]$$

$$5.41 \geq -30.34 + 49.62q \tag{24}$$

$$q \leq 0.72 \tag{25}$$

This means, when $q \leq 0.72$, Country X's best response is to invest, i.e., play $p = 1$. On the flip side, when $q \geq 0.72$, Country X's best response is to not invest, i.e., play $p = 0$.

I could go through the same motions of finding Country Y's best response to p, but because the game is symmetrical, the best response functions are flipped. So when $p \leq 0.72$, Country Y's best response is to invest, i.e., play $q = 1$. When

$p \geq 0.72$, Country Y's best response is to not invest, i.e., play $q = 0$. Country X and Country Y's best response curves are plotted in Figure 6 below.

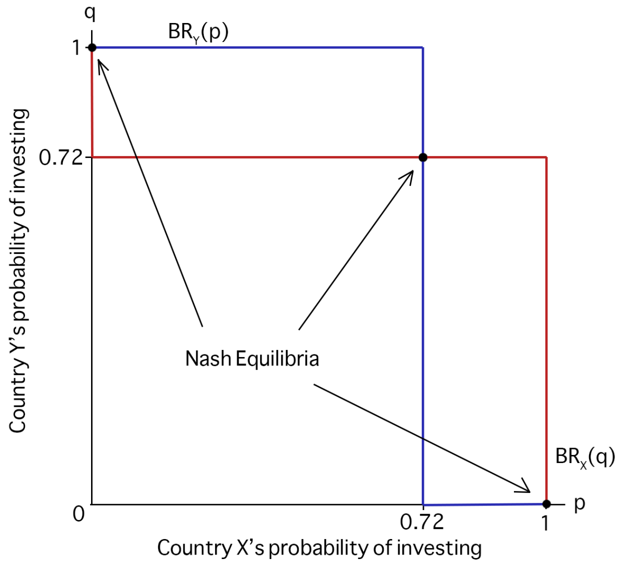

Figure 7: Mapping out the BR functions for Country X and Country Y. Nash Equilibria occur where the BR functions intersect.

In Figure 7 we can see all three Nash Equilbria. The two pure NE are (0,1) and (1,0), in which one country stands firm (doesn't invest) and the other country yields (invests). In the mixed strategy NE, each country will invest with probability 0.72. While there is merit in recognizing the existence of a mixed strategy NE, given that investment is a one-time decision and this static game is not repeated, it doesn't make sense to use welfare outcomes from the mixed strategy as a comparison to the welfare outcomes from the optimal model. Therefore, I will use the welfare outcomes from possibility 1, i.e. the pure NE where Country X invests and Country Y doesn't. This allows me to make a better comparison to the optimal model.

4.2.b Solving the Optimal Model

Similar to before, the optimal welfare function, W_T is simply the sum of W_x and W_y from the strategic model, assuming that $i_x > i_y$. The single-decision-maker will maximize the sum of Country X and Country Y's welfare. Since there is only one player, this is a decision theory model.

$$W_T = 40\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}({a_{1x}}^2 + {a_{1y}}^2) - (i_x + i_y) + 0.9[40\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2 + {a_{2y}}^2}{2(1 + \frac{i_x}{2})}] \quad (26)$$

The process of backward induction is the same for solving this optimal model. From the perspective of a single-decision-maker, it doesn't make sense for Country Y to make any investment as both countries' total abatement costs in period 2 depend on Country X's investment. Since $i_y = 0$, the single-decision-maker just maximizes W_T with respect to i_x to get optimal investment level for Country X. These are the solutions of the optimal model under a scenario where there are technology spillovers, both countries are investing in similar abatement technologies, and $i_x > i_y$:

$$i_x = 25.39, i_y = 0$$
$$a_{1x} = a_{1y} = 9.18$$
$$a_{2x} = a_{2y} = 20.42$$
$$W_T = 40.27$$

Similar to the strategic model, the solution above was found under the assumption that $i_x > i_y$. There exists another pure NE where $i_y > i_x$. Unlike the strategic model where there was a possibility that both countries would invest, or that neither would invest, a single-decision-maker choosing to optimize total welfare would not make those choices. Assuming the single-decision-maker will redistribute welfare evenly between Country X and Country Y once total welfare is maximized, it doesn't matter which pure NE they choose, as it will lead to the same outcome for Country X and Country Y. Therefore, I am keeping the assumption that $i_x > i_y$ in order to compare the decision variables and welfare outcomes between the strategic model and optimal model in the next section.

4.2.c Comparing the Strategic and Optimal Models

Scenario 2: Solutions

	Strategic	Optimal
i_x	13.84	25.39
i_y	0	0
$a_{1x} = a_{1y}$	8.22	9.18
$a_{2x} = a_{2y}$	13.06	20.42
Welfare (individual)	X: 5.41 Y: 19.25	X: 20.14 Y:20.14
Welfare (total)	24.67	40.27

Table 4: The strategic and optimal solutions for Scenario 2: Technology spillovers with similar abatement technologies, assuming that Country X invests more than Country Y.

First of all, I must preface this paragraph by stating that the solutions in Table 4 are from the pure NE where $i_x > i_y$. In both strategic and optimal models, it makes sense that $i_y = 0$ as Country Y's investment will have no impact on abatement costs in period 2, and any positive value of i_y only results in a decrease in welfare. i_x is much greater in the optimal model, suggesting that the marginal benefit of Country X's investment is greater when strategic interactions are removed.

Using a similar method of categorizing the marginal cost, marginal direct benefit, and marginal indirect benefit of investment, we can see that in this scenario, both direct and indirect marginal benefit of investment are higher for the optimal solution compared to the strategic solution. Since the marginal cost of investment is the same for W_x and W_T, and the marginal indirect benefit terms for W_x and W_T are the same as Scenario 1, I will only focus on the difference in marginal direct benefits in this solution.

The term capturing the direct benefit of i_x in W_x is $0.9[-\frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})}]$. The term capturing the direct benefit of i_x in W_T is $0.9[-\frac{{a_{2x}}^2+{a_{2y}}^2}{2(1+\frac{i_x}{2})}]$. In order to find the marginal direct benefit of i_x, I rewrite W_x and W_T as functions of i_x by substituting the strategy profile (a_{2x}^*, a_{2y}^*) into $0.9[-\frac{a_{2x}{}^2}{2(1+\frac{i_x}{2})}]$ to get:

$$\text{Direct benefit of } i_x \text{ in } W_x = \frac{9(14 + 7i_x + (2 + i_x)\sqrt{89 + 20i_x})^2}{80(1 + \frac{i_x}{2})(2 + i_x)^2} \tag{27}$$

I substitute the decision profile (a_{2x}^*, a_{2y}^*) into $0.9[-\frac{a_{2x}{}^2+a_{2y}{}^2}{2(1+\frac{i_x}{2})}]$ to get:

$$\text{Direct benefit of } i_x \text{ in } W_T = \frac{9(14 + 7i_x + (2 + i_x)\sqrt{129 + 40i_x})^2}{40(1 + \frac{i_x}{2})(2 + i_x)^2} \tag{28}$$

In order to find the marginal direct benefit of i_x in the strategic and optimal model, I take the partial derivatives of Equations (27) and (28) with respect to i_x. This gives me:

$$\text{Marginal Direct Benefit of } i_x \text{ in } W_x = \frac{63(7\sqrt{89 + 20i_x})^2}{40(2 + i_x)^2\sqrt{89 + 20i_x}} \tag{29}$$

$$\text{Marginal Direct Benefit of } i_x \text{ in } W_T = \frac{63(7\sqrt{129 + 40i_x})^2}{20(2 + i_x)^2\sqrt{129 + 40i_x}} \tag{30}$$

Looking at Equations (29) and (30), we can see that for all positive values of i_x, the marginal direct benefit of Country X's investment is greater in the optimal model. As a result, a greater marginal direct benefit in addition to a greater marginal indirect benefit of investment leads to i_x being much higher under the optimal solution.

4.3 Scenario 3: No Technology Spillovers, but Technology Can Be Bought

This scenario operates under the assumption that both countries are able to keep their technological developments secret, hence creating zero spillovers (similar to Scenario 1). However, if one country invests more than the other, they can sell their superior abatement technology to the other country for a price, p. Similar to Scenario 2, both countries are investing in similar abatement technologies. The country with the lower investment can choose to buy the "cost modifier" term from the country that invested more.

There are four possibilities: 1) Country X invests more than Country Y; Country Y buys Country X's technology, 2) Country Y invests more than Country X; Country X buys Country Y's technology, 3) Both countries invest the same amount so there is no point in buying or selling technology, and 4) Neither country invests because they assume the other country will invest.

Assuming that Country X invests more than Country Y (possibility 1), both countries will be using the "cost modifier" $\frac{1}{1+\frac{i_x}{2}}$. Thus, Country X's TAC in period 2 is $-\frac{a_{2x}{}^2}{2(1+\frac{i_x}{2})}$ and Country X's TAC in period 2 is $-\frac{a_{2y}{}^2}{2(1+\frac{i_x}{2})}$. In addition, Country X will have the addition of $+p$ in period 2 while Country Y will have the addition of $-p$ in period 2.

4.3.a Solving the Strategic Model

$$W_x = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1x}}^2 - i_x + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1+\frac{i_x}{2})} + p] \tag{31}$$

$$W_y = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1y}}^2 - i_y + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2y}}^2}{2(1+\frac{i_x}{2})} - p] \tag{32}$$

The process of solving for the strategic model is extremely similar to Scenario 2, with an additional step. Although p is decided in period 2, it is also a constant, and differentiating an individual country's welfare function with respect to p would not lead to a best response function for p. Since none of the other decision variables interact with p, I was able to use backward induction in the same way I did for Scenario 2. I followed the same method to find optimal values for investment and abatement in period 1 and 2, and then substituted these values back into W_x and W_y so that I had Country X and Country Y's welfare functions only in terms of p.

$$W_x = 0.9(6.0131 + p) \tag{33}$$

$$W_y = -19.2537 - 0.9p \tag{34}$$

From how the model is set up, being able to sell technology in period 2 doesn't make Country X more likely to invest in period 1. In order for Country X to be willing to sell their technology for price p, their welfare has to be at least greater than their welfare from not selling. This means that $p \geq 0$. If Country Y were to make a take-it-or-leave it offer to Country X, in order to maximize their own welfare they would offer $p = 0$ since Country X would be indifferent between selling and not selling their technology. Since Country Y is essentially paying a free price for Country X's technology, the individual payoffs would be the same as Scenario 2. Country Y has the higher welfare since it is still essentially "freeloading" off of Country X.

$$W_y(\text{Buy technology}) \geq W_y(\text{Do own investment})$$

$$-19.2537 - 0.9p \geq 5.41 \tag{35}$$

On the other hand, if Country X were making the take-it-or-leave-it offer, they would try to maximize their payoff while making sure that the price they choose leaves Country Y indifferent between two options. Country Y's alternative to buying technology from Country X would be to do their own investment ($i_y = 13.84$). In order for Country Y to choose zero investment and buy technology from Country X, its welfare has to be at least greater than 5.41 (Country Y's welfare from investing 13.84). By solving the inequality in line (35), we can figure out the maximum value of p that will keep Country Y indifferent between buying technology or doing their own investment. When Country X is making the take-it-or-leave-it offer, they will offer p=15.38. Total welfare doesn't change, but welfare is instead redistributed between the countries, so that Country X and County Y's payoffs are reversed.

I use the assumption that the country investing more is the one making the take-it-or-leave-it offer since this produces a set of welfare outcomes that are different from the strategic outcomes in Scenario 2. These are the solutions of the strategic model under a scenario where there are no technology spillovers, there is a duplication of investment efforts, and Country Y can buy technology from Country X in period 2 for a price of 15.38:

$$i_x = 13.84, i_y = 0$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{2x} = a_{2y} = 13.06$$
$$W_x = 19.25, W_y = 5.41$$

Similar to Scenario 2, this is the solution under the assumption that $i_x > i_y$. However, there are still three possible outcomes. The game tree for Scenario 3 is very similar to Scenario 2, except for the fact that the country that invests will have greater welfare compared to the country that doesn't invest and buys technology. Therefore, I will jump straight into the static game for the Scenario 3 strategic model.

		Country X	
		$i_x = 13.84$	$i_x = 0$
Country Y	$i_y = 13.84$	5.41, 5.41	19.25, 5.41
	$iy = 0$	5.41, 19.25	-30.34, -30.34

Table 5: Rewriting the Scenario 3 strategic model as a static game.

Unlike Scenario 2, I can see that both Country X and Country Y have a weakly dominant strategy. For both countries, choosing to invest will provide at least the same payoff if the other country invests, and will be strictly greater if the other country does not invest. Thus, the single pure Nash Equilibrium in this static game is (invest, invest). Substituting $i_x = i_y = 13.84$ into W_x and W_x gives us the final solutions of the strategic model under Scenario 3:

$$i_x = i_y = 13.84$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{2x} = a_{2y} = 13.06$$
$$W_x = W_y = 5.41$$

4.3.b Solving the Optimal Model

$$W_T = 40\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}\left(a_{1x}{}^2 + a_{1y}{}^2\right) - (i_x + i_y) + 0.9[40\ln(a_{2x} + a_{2y} - 14) - \frac{a_{2x}{}^2 + a_{2y}{}^2}{2(1 + \frac{i_x}{2})} + p - p] \quad (36)$$

The solution for the optimal model in Scenario 3 is exactly the same as the solution for the optimal model in Scenario 2. This is because the $+p$ in W_x and the $-p$ in W_y are cancelled out when summed together in W_T. Hence the solutions (under the assumption $i_x > i_y$) are still:

$$i_x = 25.39, i_y = 0$$
$$a_{1x} = a_{1y} = 9.18$$

$$a_{2x} = a_{2y} = 20.42$$

$$W_T = 40.27$$

Just like the solution in Scenario 2, there exists one more pure NE, where $i_y > i_x$. Assuming the single-decision-maker will redistribute welfare evenly between Country X and Country Y once total welfare is maximized, it doesn't matter which NE the single-decision-maker chooses, as it will lead to the same outcome for Country X and Country Y.

4.3.c Comparing the Strategic and Optimal Models

Scenario 3: Solutions

	Strategic	Optimal
i_x	13.84	25.39
i_y	13.84	0
a_{1x}=a_{1y}	8.22	9.18
a_{2x}=a_{2y}	13.06	20.42
Welfare (individual)	X: 5.41 Y: 5.41	X: 20.14 Y:20.14
Welfare (total)	10.82	40.27

Table 6: The strategic and optimal solutions for Scenario 3: No Technology spillovers but technology can be bought for $p = 15.38$*, assuming that Country X invests more than Country Y.*

Scenario 3 is interesting because total investment is actually greater in the strategic model compared to the optimal model. However, the combined welfare in the optimal model is much greater. This is because Country X and Country Y are investing in similar technologies, resulting in a duplication of efforts. This creates a lot of inefficiency. On the other hand, if the strategic incentives were removed such as in the optimal solution, the best decision would be to have one player (Country X), do all the investment so that both countries benefit from the higher investment in period 2. Because total welfare is distributed evenly in the optimal model, Country X does not incur more costs than Country Y by being the only investor.

The differences between strategic and optimal marginal benefit of investment are the same as Scenario 2. Both direct and indirect marginal benefit are greater in the optimal model.

4.4 Scenario 4: There are Technology Spillovers; Both Countries Invest in Different Abatement Technologies

This scenario operates under the assumption that there will be technology spillovers between the two countries. In contrast to Scenario 2, both countries are investing in different abatement technologies. For example, Country X invests only in renewable wind energy while Country Y invests only in renewable solar energy. The assumption I'm using for technology spillovers in period 4 is quite generous and assumes that different forms of abatement technology can be easily transferred and adopted. Because both countries cannot prevent spillovers, Country X will have the benefit of using Country Y's solar technology in addition to their own wind technology in period 2. The same is true for Country Y; Country Y will have the ability to use Country X's renewable wind technology in addition to their own solar technology in period 2.

4.4.a Solving the Strategic Model

$$W_x = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1x}}^2 - i_x + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2}{2(1 + \frac{i_x+i_y}{2})}] \quad (37)$$

$$W_y = 20\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}{a_{1y}}^2 - i_y + 0.9[20\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2y}}^2}{2(1 + \frac{i_x+i_y}{2})}] \quad (38)$$

The symmetrical set up of the strategic game in Scenario 4 is most similar to the set up of Scenario 1 (no technology spillovers). The only difference between Scenario 1 and Scenario 4 are the ”cost modifiers”. In Scenario 1, the cost modifier of each country is a function of how much investment they individually made in period 1. In Scenario 4, the cost modifier of each country is a function of how much collective investment was made in period 1. The way that technology spillovers are modeled makes W_x and W_y symmetrical, so the process of backward induction is straightforward. Since backward induction has been demonstrated extensively already, I will just display the strategic model solutions for Scenario 4:

$$i_x = i_y = 6.92$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{2x} = a_{2y} = 13.06$$
$$W_x = W_y = 12.33$$

4.4.b Solving the Optimal Model

$$W_T = 40\ln(a_{1x} + a_{1y} - 14) - \frac{1}{2}({a_{1x}}^2 + {a_{1y}}^2) - (i_x + i_y) + 0.9[40\ln(a_{2x} + a_{2y} - 14) - \frac{{a_{2x}}^2 + {a_{2y}}^2}{2(1 + \frac{i_x+i_y}{2})}] \quad (39)$$

Once again, W_T is just the sum of W_x and W_y. Similar to the last subsection, I use backward induction to solve for all six decision variables in this decision theory model. Just like the optimal model in Scenario 1, the best response functions for i_x and i_y overlapped perfectly, meaning the single-decision-maker has an infinite number of combinations of i_x and i_y that maximize total welfare. Assuming that the single-decision-maker wants to diversify the investments as much as possible, they will choose the solution where $i_x = i_y$, which produces the solutions:

$$i_x = i_y = 12.70$$
$$a_{1x} = a_{1y} = 9.18$$
$$a_{2x} = a_{2y} = 20.42$$
$$W_T = 40.27$$

4.4.c Comparing the Strategic and Optimal Models

Similar to the solutions of Scenario 1, both the strategic and optimal solutions are symmetric. The optimal solution is Pareto superior compared to the strategic solution as it leads to greater total welfare and greater individual welfare for both countries. The individual investment in the optimal solution is much greater than the individual investment in the

Scenario 4: Solutions

	Strategic	Government
i_x=i_y	6.92	12.70
a_{1x}=a_{1y}	8.22	9.18
a_{2x}=a_{2y}	13.06	20.42
Welfare (individual)	X: 12.33 Y: 12.33	X: 20.14 Y: 20.14
Welfare (total)	24.67	40.27

Table 7: The strategic and optimal solutions for Scenario 4: There are technology spillovers; Country X and Country Y are investing in different abatement technologies.

strategic solution. Given that the cost of individual investment is the same in W_x and W_T, it must be differences in the marginal benefit of investment. The term capturing direct benefit of Country X's investment in the strategic model is $0.9[-\frac{{a_{2x}}^2}{2(1+\frac{i_x+i_y}{2})}]$. The term capturing direct benefit of Country X's investment in the optimal model is $0.9[-\frac{{a_{2x}}^2+{a_{2y}}^2}{2(1+\frac{i_x+i_y}{2})}]$. I first rewrite these expressions as functions of i_x and i_y. I substitute the strategy profile (a^*_{2x}, a^*_{2y}) into $0.9[-\frac{a_{2x}}{2(1+\frac{i_x+i_y}{2})}]$ to get:

$$\text{Direct benefit of } i_x \text{ in } W_x = -\frac{9(7+\sqrt{89+20i_x+20i_y})^2}{40(2+i_x+i_y)} \tag{40}$$

I substitute the decision profile (a^*_{2x}, a^*_{2y}) into $0.9[-\frac{{a_{2x}}^2+{a_{2y}}^2}{2(1+\frac{i_x+i_y}{2})}]$ to get:

$$\text{Direct benefit of } i_x \text{ in } W_x = -\frac{9(7+\sqrt{129+40i_x+40i_y})^2}{20(2+i_x+i_y)} \tag{41}$$

By taking the derivative of Equations (40) and (41) with respect to i_x, I get the marginal direct benefit of Country X's investment in the strategic and optimal model.

$$\text{Marginal Direct Benefit of } i_x \text{ in } W_x = \frac{63(7\sqrt{89+20i_x+20i_y})^2}{40(2+i_x+i_y)^2\sqrt{89+20i_x+20i_y}} \tag{42}$$

$$\text{Marginal Direct Benefit of } i_x \text{ in } W_T = \frac{63(7\sqrt{129+40i_x+40i_y})^2}{20(2+i_x+i_y)^2\sqrt{129+40i_x+40i_y}} \tag{43}$$

Looking at Equations (42) and (43), I can see that for all positive values of i_x and i_y, the marginal direct benefit of i_x will be greater in the optimal welfare function. I can go through the same process to find the marginal indirect benefit of i_x in W_x and W_T. The indirect benefit of investment is the greater welfare resulting from higher collective abatement levels due to lower total abatement cost in period 2. It is the $0.9[20\ln(a_{2x}+a_{2y}-14)]$ term in W_x and the $0.9[40\ln(a_{2x}+a_{2y}-14)]$ term in W_T. Since these terms are equal to the indirect benefit terms of W_x and W_T from Scenario 1, there is no need to walk through the process of finding the marginal indirect benefit again. We know from Scenario 1 that the indirect marginal benefit of investment is greater in the optimal model compared to the strategic model.

To conclude, both marginal direct benefit and marginal indirect benefit of investment is greater in the optimal model, resulting in higher investment in the optimal solution.

5 Discussion and Conclusion

5.1 A Comparison of Solutions Under the Four Scenarios

	1: No technology spillovers		2. Spillovers, similar abatement technologies		3. No spillovers, but technology can be bought		4. Spillovers, different abatement technologies	
	Strategic	Optimal	Strategic	Optimal	Strategic	Optimal	Strategic	Optimal
i_x	1.67	13.64	13.84	25.39	13.84	25.39	6.92	12.70
i_y	1.67	13.64	0	0	13.84	0	6.92	12.70
$a_{1x} = a_{1y}$	8.22	9.18	8.22	9.18	8.22	9.18	8.22	9.18
$a_{2x} = a_{2y}$	9.03	16.49	13.06	20.42	13.06	20.42	13.06	20.42
Welfare (individual)	X: -12.39 Y: -12.39	X: 11.00 Y: 11.00	X: 5.41 Y: 19.25	X: 20.14 Y: 20.14	X: 5.41 Y: 5.41	X: 20.14 Y: 20.14	X: 12.33 Y: 12.33	X: 20.14 Y: 20.14
Welfare (total)	-24.77	22.01	24.67	40.27	10.82	40.27	24.67	40.27

Table 8: The strategic and optimal solutions for all four scenarios.

5.1.a General Observations

Across all four scenarios, the optimal models result in Pareto superior outcomes compared to the strategic models. This is not surprising considering all four scenarios contain actions with positive externalities, whether it is carbon emissions abatement, or technology spillovers.

In addition, the values of a_{1x} and a_{1y} are 8.22 in all four strategic scenarios, while the values of a_{1x} and a_{1y} are 9.18 in all four optimal scenarios. This is expected since abatement costs and benefits in period 1 are not influenced by the presence of technology spillovers. a_{1x} and a_{1y} are understandably greater in the optimal solutions because the marginal benefit of abatement is greater for a single-decision-maker.

With a_{2x} and a_{2y}, we see a similar pattern for Scenarios 2, 3 and 4. $a_{2x} = a_{2y} = 13.06$ for the strategic solutions while $a_{2x} = a_{2y} = 20.42$ for the optimal solutions. This demonstrates that despite differences in investment spending, the period 2 optimal abatement level is the same when any form of technology transfer occurs, whether it comes from spillovers or payment.

When technology spillovers happen under the assumption of a duplication of investment efforts (Scenario 2 and Scenario 3), the Nash Equilibrium is an asymmetrical solution where one country invests and the other does not. This is apparent in the strategic and optimal solutions for Scenario 2, and the optimal solution for Scenario 3. The exception to this is the strategic solution from Scenario 3, when we assume that the investing country makes the take-it-or-leave it offer. As a result, the weakly dominant strategy for both countries is to invest, leading to inefficiency. Thus, while the strategic solution under Scenario 3 may seem more ”fair” because $W_x = W_y$, it actually leads to lower total welfare compared to the strategic solution under Scenario 2.

Out of all the solutions, the optimal models under Scenario 2, 3 and 4 produced the greatest total welfare: 40.27. This suggests that it doesn’t matter whether it was a spillover or purchase, technology transfer under the guidance of a single-decision-maker will lead to a Pareto superior outcome. Out of the four strategic solutions, Scenario 2 and 4 produced the greatest total welfare: 24.67. Though the total welfare is the same, welfare is distributed unevenly in Scenario 2's strategic solution, while $W_x = W_y$ in Scenario 4's strategic solution.

5.2 Key Takeaways in a Real world Context

For the most part, technology spillovers lead to increased carbon emissions abatement in future periods. Though the positive externalities of technology spillovers lead to sub-optimal welfare outcomes in strategic interactions, it is still better than a world with zero spillovers. Looking at Table 8, we see that the optimal solution in a world with no technology spillovers (Scenario 1), is worse than strategic solutions in a world with technology spillovers (Scenario 2 and 4). In the real world, it is impossible to prevent zero technology spillovers. Therefore, Scenario 2 and 4 are more realistic descriptions of our reality.

Though the single-decision-maker solutions lead to the best outcomes across all four scenarios, these outcomes would be a result of something akin to an international government organization that has the power to force each country to invest and abate a certain amount. From historical failures such as the Kyoto Protocol to current debates about the Paris Agreement, it seems highly unlikely that countries will start cooperating in a manner that leads to optimal welfare outcomes anytime soon.

Therefore, it seems more productive to use the strategic solutions from Scenario 2 and 4 to draw insights for potential real world outcomes. Scenario 2 and 4 are similar in that both involve technology spillovers. However, in Scenario 2, there is a duplication of investment efforts. On the other hand, in Scenario 4, the countries are investing in different technologies, so the the spillover effect is greater. Both of these hypothetical scenarios have their own merits in describing the real world.

Despite the total welfare outcomes being the same in Scenario 2 and 4, Scenario 4 arguably has better individual outcomes because investment, abatement and welfare are split evenly between both countries. On the other hand, in Scenario 2, the country that invests bears the burden of doing all the investment, which makes the individual outcomes unfair. As a reminder, the strategic solution for Scenario 2 shown in Table 8 is just one of three Nash Equilibria. At its root, Scenario 2 is an anti-coordination game and the outcome of 24.67 as the total welfare is not guaranteed. Due to how Scenario 4 is set up, both countries have incentives to invest, and its strategic solution in Table 8 is the only Nash Equilibrium. So, theoretically, a world in which the total cost of abatement is a function of collective investment will give us the best outcome.

At the same time, I have to be careful of relying too heavily on these game theoretic models as they are grossly simplified versions of the real world. There are two major issues with the assumptions of Scenario 4. First, In the real world, countries don't just invest in one type of abatement technology, or deliberately invest in technologies that are distinctly different from what other countries are investing in. Also, carbon emissions abatement can be achieved in a variety of ways. Investing in abatement technologies is a very narrow category of all the actions that fall under the umbrella of "climate mitigation". Countries will make different types of investment in order to diversify and spread risk. After all, the relationships linking investment, technological innovation, and technological improvements are nonlinear and involve some aspect of uncertainty. However, the key takeaway is that Scenario 4 suggests that some specialization in investment could be beneficial if this leads to a greater range of abatement technologies for all countries to adopt in the future. The second issue with Scenario 4 is its assumption that different forms of technology can be easily transferred with 100 percent efficiency. Obviously, this is not true; even if technology such as renewable solar energy was freely transferred, it would not be effective at reducing emissions in countries that don't have the right climate to harness solar energy.

Though Scenario 4 produces more equitable individual welfare outcomes, Scenario 2 might be the more accurate (though still too simplistic) representation of the real world. Like I mentioned above, countries don't just invest in one type of abatement technology. "Abatement technology" includes a diverse range of mitigation actions that include reforestation, solar energy, biomass, improving fuel economy, waste recycling and much more. In the real world, most countries are engaged in a wide range of emissions abatement activities. We see a lot of countries investing in the same technologies, so this seems most similar to Scenario 2, where we have a duplication of investment efforts. This doesn't bode too well, because Scenario 2 is an anti-coordination game.

5.3 Final Thoughts

Overall, the model that I propose in this paper is a very simplistic model. The "carbon threshold" of 15 gigatonnes of cumulative emissions abatement every time period is the only aspect truly grounded in empirical data. The other parameters in this model were chosen such that it would be possible to optimize the welfare function and find viable solutions. This is a theoretical model full of hypotheticals. Perhaps in reality, the return on investment is extremely low and we should just put all our efforts into mitigation actions right now. Furthermore, though asymmetry is worked into the model due to technology spillovers, the baseline welfare functions of Country X and Country Y are still symmetrical. This seems to imply that the "fair" way to spread out welfare is to divide it evenly. A symmetrical model doesn't take into account that one country may be more vulnerable to climate change than the other, or that one country may be the source of most of the historical emissions and should bear a higher cost of mitigation. Lastly, since this is a two-player game, my model does not capture the complexity that arises when you are not just playing a best response to one player, but multiple players.

Despite all these limitations, my model does achieve what I set out to do, which was to explore what happens to abatement levels and investment in the presence of technology spillovers. A world with technology spillovers in which each country acts in their own best self interest is still better than a world in which every country acts optimally but does not share technological advances. And while it makes sense for countries to diversify their abatement methods, some degree of specialization in different abatement technologies may lead to greater spillovers for all countries in the future.

6 Appendix

A Solving the Strategic Model

Using backwards induction, the very first step is the find the best response functions of a_{2x} and a_{2y} in terms of i_x and i_y so that I can find the strategy profile (a^*_{2x}, a^*_{2y}).

$$\frac{\partial W_x}{\partial a_{2x}} = \frac{18}{14 + a_{2x} + a_{2y}} - \frac{9a_{2x}}{5(2 + i_x)} \tag{44}$$

I set the partial derivative equal to zero.

$$0 = \frac{18}{14 + a_{2x} + a_{2y}} - \frac{9a_{2x}}{5(2 + i_x)} \tag{45}$$

I rearrange line (45) in order to get the best response function of a_{2x}; there are actually two possible BR functions:

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} - \sqrt{276 - 28a_{2y} + {a_{2y}}^2 + 40i_x}\right) \tag{46}$$

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} + \sqrt{276 - 28a_{2y} + {a_{2y}}^2 + 40i_x}\right) \tag{47}$$

I do the same steps in (44) and (45) to get two BR functions for a_{2y}:

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} - \sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}\right) \tag{48}$$

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} + \sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}\right) \tag{49}$$

Since W_x and W_y are completely symmetrical, I assume that either the strategy profiles for a_{2x} and a_{2y} are both the positive root, or both the negative root. The next step into the substitute the BR functions into each other, since the point where they intersect is the strategy profile that is played. I try the negative root solutions first, substituting (48) into the LHS of (46), and solve.

$$\begin{aligned} a^*_{2x} = \frac{1}{4}&\left(14 + a_{2x} + \sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}\right) \\ &- \frac{1}{4}\left(\sqrt{2}\sqrt{396 + {a_{2x}}^2 + 80i_x + 20i_y + (14 + a_{2x})\sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}}\right) \end{aligned} \tag{50}$$

$$a^*_{2x} = \frac{14 + 7i_x \pm \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y} \tag{51}$$

I then try the positive root solutions, substituting (49) into the LHS of Equation 47, and solve.

$$a_{2x}^* = \frac{1}{4}\left(14 + a_{2x} - \sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}\right) + \frac{1}{4}\left(\sqrt{2}\sqrt{396 + {a_{2x}}^2 + 80i_x + 20i_y - (14 + a_{2x})\sqrt{276 - 28a_{2x} + {a_{2x}}^2 + 40i_y}}\right) \tag{52}$$

$$a_{2x}^* = \frac{14 + 7i_x \pm \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y} \tag{53}$$

The solutions (51) and (53) are actually the same. So a_{2x}^* is the same, using either the positive root or negative root . Going through these same steps, I arrive at Country Y's strategy for a_{2y}:

$$a_{2y}^* = \frac{14 + 7i_y \pm \sqrt{(2 + i_y)^2(89 + 10i_y + 10i_x)}}{4 + i_x + i_y} \tag{54}$$

Now that we have the strategy profile (a_{2x}^*, a_{2y}^*), it's time to substitute it back into W_x and W_y. Both a_{2x}^* and a_{2y}^* have a positive and negative root, so I will first substitute the positive root, $\frac{14+7i_y+\sqrt{(2+i_y)^2(89+10i_y+10i_x)}}{4+i_x+i_y}$ into the welfare functions, but I will come back and redo these steps with the negative root, $\frac{14+7i_y-\sqrt{(2+i_y)^2(89+10i_y+10i_x)}}{4+i_x+i_y}$, as well.

$$W_x = -\frac{{a_{1x}}^2}{2} - i_x + 20\ln(-14 + a_{1x} + a_{1y}) - \frac{9\left(14 + 7i_x + \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)}\right)^2}{10(2 + i_x)(4 + i_x + i_y)^2} + 18\ln\left(\frac{-28 - 7i_x - 7i_y + \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)} + \sqrt{(2 + i_y)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y}\right) \tag{55}$$

$$W_y = -\frac{{a_{1y}}^2}{2} - i_y + 20\ln(-14 + a_{1x} + a_{1y}) - \frac{9\left(14 + 7i_y + \sqrt{(2 + i_y)^2(89 + 10i_x + 10i_y)}\right)^2}{10(2 + i_y)(4 + i_x + i_y)^2} + 18\ln\left(\frac{-28 - 7i_x - 7i_y + \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)} + \sqrt{(2 + i_y)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y}\right) \tag{56}$$

The next step is to find BR functions for a_{1x} and a_{1y}, in order to find the strategy profile (a_{1x}^*, a_{1y}^*). I take the partial derivatives of W_x and W_y with respect to a_{1x} and a_{1y}, respectively.

$$\frac{\partial W_x}{\partial a_{1x}} = -a_{1x} + \frac{20}{a_{1x} + a_{1y} - 14} \tag{57}$$

I then set the partial derivative equal to zero and solve to get two functions:

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}(14 - a_{1y} - \sqrt{276 - 28a_{1y} + {a_{1y}}^2}) \tag{58}$$

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}(14 - a_{1y} + \sqrt{276 - 28a_{1y} + a_{1y}{}^2}) \tag{59}$$

I do the same steps and arrive at the two BR functions for a_{1y}.

$$BR_{a_{1t}}(a_{1x}) = \frac{1}{2}(14 - a_{1x} - \sqrt{276 - 28a_{1x} + a_{1x}{}^2}) \tag{60}$$

$$BR_{a_{1t}}(a_{1x}) = \frac{1}{2}(14 - a_{1x} + \sqrt{276 - 28a_{1x} + a_{1x}{}^2}) \tag{61}$$

Due to the symmetry of the strategic interaction model, I assume that a_{1x}=a_{1y}, so when I'm substituting the BR functions into each other, I am using either both the positive roots, or both the negative roots. First I try the negative root. I substitute (60) into the LHS of (58), and solve for the value of a^*_{1x}.

$$a^*_{1x} = \frac{1}{4}\left(14 + a_{1x} + \sqrt{276 - 28a_{1x} + a_{1x}{}^2} - \sqrt{2}\sqrt{396 + a_{1x}{}^2 + (14 + a_{1x})\sqrt{276 - 28a_{1x} + a_{1x}{}^2}}\right) \tag{62}$$

$$a^*_{1x} = \frac{1}{2}(7 - \sqrt{89}) \tag{63}$$

I then try the positive root. I substitute (61) into the LHS of (59) and solve for the value of a^*_{1x}.

$$a^*_{1x} = \frac{1}{4}\left(14 + a_{1x} - \sqrt{276 - 28a_{1x} + a_{1x}{}^2} + \sqrt{2}\sqrt{396 + a_{1x}{}^2 - (14 + a_{1x})\sqrt{276 - 28a_{1x} + a_{1x}{}^2}}\right) \tag{64}$$

$$a^*_{1x} = \frac{1}{2}(7 + \sqrt{89}) \tag{65}$$

Similar to above, the strategy for a^*_{1x} has a negative and positive solution. However, knowing that $20\ln(a_{1x} + a_{1y} - 14)$ is a component of the welfare function, the sum of a_{1x} and a_{1y} must be greater than 14, otherwise the function is undefined. a^*_{1x} in (63) is a negative number, so it cannot be a valid solution for a_{1x}. Therefore, $a^*_{1x} = \frac{1}{2}(7 + \sqrt{89})$ is the valid solution. Since we know that the game is symmetrical and $a^*_{1x} = a^*_{1y}$:

$$a^*_{1x} = a^*_{1y} = \frac{1}{2}(7 + \sqrt{89}) \tag{66}$$

The next step is to substitute the strategy profile (a^*_{1x}, a^*_{1y}) back into W_x and W_y.

$$\begin{aligned} W_x = &-\frac{1}{8}(7 + \sqrt{89})^2 - i_x + 20\ln\left[-7 + \sqrt{89}\right] - \frac{9\left(14 + 7i_x + \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)}\right)^2}{10(2 + i_x)(4 + i_x + i_y)^2} \\ &+ 18\ln\left[\frac{-28 - 7i_x - 7i_y + \sqrt{(2 + i_x)^2(89 + 10i_x + 10i_y)} + \sqrt{(2 + i_y)^2(89 + 10i_x + 10i_y)}}{4 + i_x + i_y}\right] \end{aligned} \tag{67}$$

$$W_y = -\frac{1}{8}(7+\sqrt{89})^2 - i_y + 20\ln\left[-7+\sqrt{89}\right] - \frac{9\left(14+7i_y+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_y)(4+i_x+i_y)^2}$$
$$+ 18\ln\left[\frac{-28-7i_x-7i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}\right] \quad (68)$$

Now that we have W_x and W_y in terms of just i_x and i_y, I will find the BR functions of i_x and i_y by taking the partial derivative of W_x and W_y with respect to i_x and i_y, respectively.

$$\frac{\partial W_x}{\partial i_x} = -1 + \frac{9\left(14+7i_x+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{5(2+i_x)(4+i_x+i_y)^3} + \frac{9\left(14+7i_x+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_x)^2(4+i_x+i_y)^2}$$
$$-\frac{9\left(14+7i_x+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)\left((2+i_y)(99+10i_x+15i_y)+7\sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)}{5(2+i_x)(4+i_x+i_y)^2\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}$$
$$-\frac{18\left(28+7i_x+i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}{(4+i_x+i_y)\left(-28-7i_x-7i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}$$
$$+\frac{18(4+i_x+i_y)\left(\frac{5(2+i_y)^2}{\sqrt{(2+i_y)^2(89+10i_x+10i_y)}}+\frac{(2+i_x)(99+10i_y+15i_x)+7\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}{\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}\right)}{(4+i_x+i_y)\left(-28-7i_x-7i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)} \quad (69)$$

$$\frac{\partial W_y}{\partial i_y} = -1 + \frac{9\left(14+7i_y+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{5(2+i_y)(4+i_x+i_y)^3} + \frac{9\left(14+7i_y+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_y)^2(4+i_x+i_y)^2}$$
$$-\frac{9\left(14+7i_y+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)\left((2+i_x)(99+15i_x+10i_y)+7\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}{5(2+i_y)(4+i_x+i_y)^2\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}$$
$$-\frac{18\left(28+7i_x+i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}{(4+i_x+i_y)\left(-28-7i_x-7i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}$$
$$+\frac{18(4+i_x+i_y)\left(\frac{5(2+i_x)^2}{\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}+\frac{(2+i_y)(99+10i_x+15i_y)+7\sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{\sqrt{(2+i_y)^2(89+10i_x+10i_y)}}\right)}{(4+i_x+i_y)\left(-28-7i_x-7i_y+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}+\sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)} \quad (70)$$

The partial derivative terms are very complex. At this point, the next step is to set $\frac{\partial W_x}{\partial i_x}$ and $\frac{\partial W_y}{\partial i_y}$ equal to zero, in order to find the BR functions for i_x and i_y. However, this was computationally too difficult for Mathematica to do. Instead, I used the `ContourPlot` command in Mathematica to plot $\frac{\partial W_x}{\partial i_x} = 0$ and $\frac{\partial W_y}{\partial i_y} = 0$ as level curves on the same graph. Having the BR functions graphed out and finding their intersection point is an alternative method of finding the strategy profile (i_x^*, i_y^*) to the previously used method of substituting the BR functions into each other.

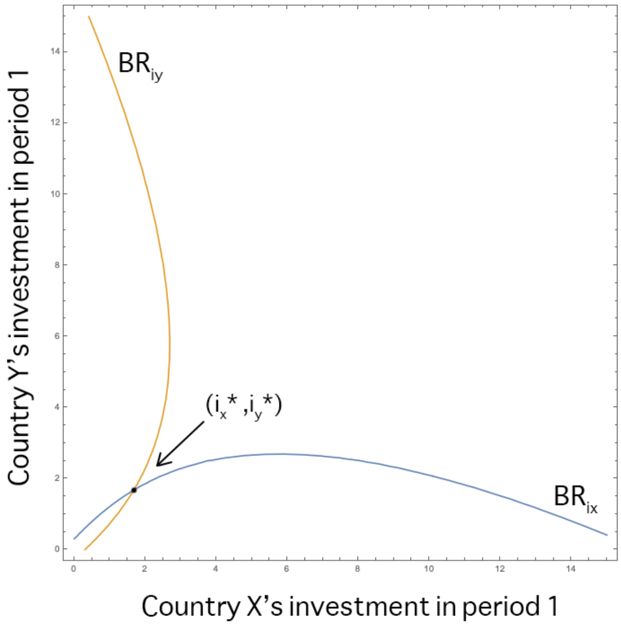

Figure 8: Graphing the level curves $\frac{\partial W_x}{\partial i_x} = 0$ and $\frac{\partial W_y}{\partial i_y} = 0$ to find the strategy profile (i_x^*, i_y^*). Figure 8 is the same as Figure 4 from subsection 4.1.a.

I use the `Apply@RegionIntersection` command in Mathematica to find the point of intersection, which is (1.67325, 1.67325). Now I have the strategy profile of (i_x^*, i_y^*). I substitute (1.67325, 1.67325) back into the strategy profile (a_{2x}^*, a_{2y}^*) to get the optimal values of abatement in period 2. I also substitute (1.67325, 1.67325) back into W_x and W_y to get the welfare outcomes. With that, I have the optimal values of all the decision variables and the welfare outcomes in the strategic model for Scenario 1:

$$i_x = i_y = 1.67$$
$$a_{1x} = a_{1y} = 8.22$$
$$a_{1x} = a_{1y} = 9.03$$
$$W_x = W_y = -12.39$$

Before moving onto, I have to go back and check the negative root solution of (a_{2x}^*, a_{2y}^*). In Lines (55) and (56) I substituted $a_{2x}^* = a_{2y}^* = \frac{14+7i_y+\sqrt{(2+i_y)^2(89+10i_y+10i_x)}}{4+i_x+i_y}$ into W_x and W_y. Now I will substitute $a_{2x}^* = a_{2y}^* = \frac{14+7i_y-\sqrt{(2+i_y)^2(89+10i_y+10i_x)}}{4+i_x+i_y}$ into W_x and W_y to see if it gives us any viable solutions.

$$W_x = -\frac{{a_{1x}}^2}{2} - i_x + 20\ln(-14 + a_{1x} + a_{1y}) - \frac{9\left(-14 - 7i_x + \sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_x)(4+i_x+i_y)^2} + 18\ln\left(-\frac{28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}\right) \quad (71)$$

$$W_y = -\frac{{a_{1y}}^2}{2} - i_y + 20\ln(-14 + a_{1x} + a_{1y}) - \frac{9\left(-14 - 7i_y + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_y)(4+i_x+i_y)^2}$$
$$+ 18\ln\left(-\frac{28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}\right) \quad (72)$$

The next step is to find BR functions for a_{1x} and a_{1y}, in order to find the strategy profile (a^*_{1x}, a^*_{1y}). Since the only difference between the welfare functions in (71) & (72) and (55) & (56) are the inputs for $(a^*_{2x}$ and $a^*_{2y})$, the partial derivatives for a_{1x} and a_{1y} will be the same, and so will the strategy profile, so I will skip the steps and just substitute $a^*_{1x} = a^*_{1y} = \frac{1}{2}(7 + \sqrt{89})$ back into W_x and W_y.

$$W_x = -\frac{1}{8}(7+\sqrt{89})^2 - i_x + 20\ln\left[-7+\sqrt{89}\right] - \frac{9\left(-14 - 7i_x + \sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_x)(4+i_x+i_y)^2}$$
$$+ 18\ln\left[-\frac{28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}\right] \quad (73)$$

$$W_y = -\frac{1}{8}(7+\sqrt{89})^2 - i_y + 20\ln\left[-7+\sqrt{89}\right] - \frac{9\left(-14 - 7i_y + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_y)(4+i_x+i_y)^2}$$
$$+ 18\ln\left[-\frac{28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}}{4+i_x+i_y}\right] \quad (74)$$

Now that we have W_x and W_y in terms of just i_x and i_y, I will find the BR functions of i_x and i_y by taking the partial derivative of W_x and W_y with respect to i_x and i_y, respectively.

$$\frac{\partial W_x}{\partial i_x} = -1 + \frac{9\left(-14 - 7i_x + \sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{5(2+i_x)(4+i_x+i_y)^3} + \frac{9\left(-14 - 7i_x + \sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_x)^2(4+i_x+i_y)^2}$$
$$- \frac{9\left(-14 - 7i_x + \sqrt{(2+i_x)^2(89+10i_x+10i_y)}\right)\left(-7 + \frac{(2+i_x)(99+15i_x+10i_y))}{\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}\right)}{5(2+i_x)(4+i_x+i_y)^2}$$
$$+ \frac{18\left(-1 + (4+i_x+i_y)\left(7 + \frac{5(2+i_y)^2}{\sqrt{(2+i_y)^2(89+10i_x+i_y)}+}\right) + \frac{(2+i_x)(99+15i_x+10i_y)}{\sqrt{(2+i_x)^2(89+10i_x+i_y)}}\right)}{(4+i_x+i_y)\left(28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)}$$

(75)

$$\frac{\partial W_x}{\partial i_x} = -1 + \frac{9\left(-14 - 7i_x + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{5(2+i_y)(4+i_x+i_y)^3} + \frac{9\left(-14 - 7i_x + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)^2}{10(2+i_y)^2(4+i_x+i_y)^2}$$
$$- \frac{9\left(-14 - 7i_x + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)\left(-7 + \frac{(2+i_y)(99+10i_x+15i_y))}{\sqrt{(2+i_y)^2(89+10i_x+10i_y)}}\right)}{5(2+i_y)(4+i_x+i_y)^2}$$
$$+ \frac{18\left(-1 + (4+i_x+i_y)\left(7 + \frac{5(2+i_x)^2}{\sqrt{(2+i_x)^2(89+10i_x+i_y)+}}\right) + \frac{(2+i_y)(99+10i_x+15i_y)}{\sqrt{(2+i_y)^2(89+10i_x+i_y)}}\right)}{(4+i_x+i_y)\left(28 + 7i_x + 7i_y + \sqrt{(2+i_x)^2(89+10i_x+10i_y)} + \sqrt{(2+i_y)^2(89+10i_x+10i_y)}\right)} \tag{76}$$

Similar to before, the next step would be to use the `ContourPlot` command in Mathematica to plot $\frac{\partial W_x}{\partial i_x} = 0$ and $\frac{\partial W_y}{\partial i_y} = 0$ as level curves on the same graph. However, when plotting the level curves, neither BR function appeared, suggesting that there are no values of i_x and i_y that can fulfill the requirement $\frac{\partial W_x}{\partial i_x} = 0$ or $\frac{\partial W_y}{\partial i_y} = 0$. I used the `Plot3D` command in Mathematica to plot out $\frac{\partial W_x}{\partial i_x}$, and confirmed this to be true. Therefore, the backward induction process stops here. The negative root solution for a^*_{2x} and a^*_{2y}, $\frac{14+7i_y-\sqrt{(2+i_y)^2(89+10i_y+10i_x)}}{4+i_x+i_y}$, is not a valid function as it does not lead to BR functions i_x and i_y later on. The process of checking the negative root confirms that the only valid solutions come from using $a^*_{2x} = a^*_{2y} = \frac{14+7i_x+\sqrt{(2+i_x)^2(89+10i_x+10i_y)}}{4+i_x+i_y}$. The only valid set of solutions for the strategic model under Scenario 1 are on page 33.

B Solving the Optimal Model

Using backward induction, the very first step is the find the single-decision-maker's best response functions of a_{2x} and a_{2y} in terms of i_x and i_y so that I can find the decision profile (a^*_{2x}, a^*_{2y}).

$$\frac{\partial W_T}{\partial W_x} = -\frac{9\left({a_{2x}}^2 + a_{2x}(-14+a_{2y}) - 20(2+i_x)\right)}{5(-14+a_{2x}+a_{2y})(2+i_x)} \tag{77}$$

I set the partial derivative equal to zero.

$$0 = -\frac{9\left({a_{2x}}^2 + a_{2x}(-14+a_{2y}) - 20(2+i_x)\right)}{5(-14+a_{2x}+a_{2y})(2+i_x)} \tag{78}$$

I rearrange line (78) in order to get the single-decision-maker's best response function of a_{2x}; there are two possible BR functions:

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} - \sqrt{356 - 28a_{2y} + {a_{2y}}^2 + 80i_x}\right) \tag{79}$$

$$BR_{a_{2x}}(i_x, a_{2y}) = \frac{1}{2}\left(14 - a_{2y} + \sqrt{356 - 28a_{2y} + {a_{2y}}^2 + 80i_x}\right) \tag{80}$$

I do the same steps in (77) and (78) to get two BR functions for a_{2y}:

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} - \sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}\right) \tag{81}$$

$$BR_{a_{2y}}(i_y, a_{2x}) = \frac{1}{2}\left(14 - a_{2x} + \sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}\right) \tag{82}$$

Since W_T is simply the sum of W_x and W_y, I am still expecting a symmetrical solution where a_{2x} and a_{2y} are both the positive root, or both the negative root. I try the negative root solutions first, substituting (81) into the LHS of (79), and solve.

$$\begin{aligned} a^*_{2x} = \frac{1}{4}&\left(14 + a_{2x} + \sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}\right) \\ &- \frac{1}{4}\left(\sqrt{2}\sqrt{596 + {a_{2x}}^2 + 160i_x + 40i_y + (14 + a_{2x})\sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}}\right) \end{aligned} \tag{83}$$

$$a^*_{2x} = \frac{14 + 7i_x \pm \sqrt{(2 + i_x)^2(129 + 20i_x + 20i_y)}}{4 + i_x + i_y} \tag{84}$$

I then try the positive root solutions, substituting (82) into the LHS of Equation (80), and solve.

$$\begin{aligned} a^*_{2x} = \frac{1}{4}&\left(14 + a_{2x} - \sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}\right) \\ &+ \frac{1}{4}\left(\sqrt{2}\sqrt{596 + {a_{2x}}^2 + 160i_x + 40i_y - (14 + a_{2x})\sqrt{356 - 28a_{2x} + {a_{2x}}^2 + 80i_y}}\right) \end{aligned} \tag{85}$$

$$a^*_{2x} = \frac{14 + 7i_x \pm \sqrt{(2 + i_x)^2(129 + 20i_x + 20i_y)}}{4 + i_x + i_y} \tag{86}$$

The solutions in (86) and (84) are actually the same. So a^*_{2x} is the same, using either the positive root or negative root . Going through these same steps, I arrive at Country Y's strategy for a_{2y}:

$$a^*_{2y} = \frac{14 + 7i_y \pm \sqrt{(2 + i_y)^2(129 + 20i_y + 20i_x)}}{4 + i_x + i_y} \tag{87}$$

Now that we have the single-decision-maker's decision profile (a^*_{2x}, a^*_{2y}), it's time to substitute it back into W_T. Both a^*_{2x} and a^*_{2y} have a positive and negative root, but I substitute $a^*_{2x} = a^*_{2y} = \frac{14+7i_y+\sqrt{(2+i_y)^2(129+20i_y+20i_x)}}{4+i_x+i_y}$ into W_T, because from Appendix A I know that the negative root for (a^*_{2x} and a^*_{2y}) is not going to give me a valid solution.

$$W_T = \frac{1}{10}\left(-5{a_{1x}}^2 - 5{a_{1y}}^2 - 10i_x - 10i_y + 400\ln[-14 + a_{1x} + a_{1y}]\right)$$
$$+\frac{9}{10}\left(-\frac{\left(14+7i_x+\sqrt{(2+i_x)^2(129+20i_x+20i_y)}\right)^2}{(2+i_x)(4+i_x+i_y)^2} - \frac{\left(14+7i_y+\sqrt{(2+i_y)^2(129+20i_x+20i_y)}\right)^2}{(2+i_y)(4+i_x+i_y)^2}\right)$$
$$+36\ln\left[\frac{-28-7i_x-7i_y+\sqrt{(2+i_x)^2(129+20i_x+20i_y)}+\sqrt{(2+i_y)^2(129+20i_x+20i_y)}}{4+i_x+i_y}\right] \quad (88)$$

The next step is to find the single-decision-maker's BR functions for a_{1x} and a_{1y}, in order to find the decision profile (a^*_{1x}, a^*_{1y}). I take the partial derivatives of W_T with respect to a_{1x} and a_{1y}.

$$\frac{\partial W_T}{\partial a_{1x}} = -a_{1x} + \frac{40}{a_{1x} + a_{1y} - 14} \quad (89)$$

$$\frac{\partial W_T}{\partial a_{1y}} = -a_{1y} + \frac{40}{a_{1x} + a_{1y} - 14} \quad (90)$$

I then set $\frac{\partial W_T}{\partial a_{1x}}$ equal to zero and solve to get two functions:

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}(14 - a_{1y} - \sqrt{356 - 28a_{1y} + {a_{1y}}^2}) \quad (91)$$

$$BR_{a_{1x}}(a_{1y}) = \frac{1}{2}(14 - a_{1y} + \sqrt{356 - 28a_{1y} + {a_{1y}}^2}) \quad (92)$$

I do the same steps and arrive at the two BR functions for a_{1y}.

$$BR_{a_{1y}}(a_{1x}) = \frac{1}{2}(14 - a_{1x} - \sqrt{356 - 28a_{1x} + {a_{1x}}^2}) \quad (93)$$

$$BR_{a_{1y}}(a_{1x}) = \frac{1}{2}(14 - a_{1x} + \sqrt{356 - 28a_{1x} + {a_{1x}}^2}) \quad (94)$$

Due to the symmetry of the strategic interaction model, I assume that a_{1x}=a_{1y}, so when I'm substituting the BR functions into each other, I am using either both the positive roots, or both the negative roots. First I try the negative root. I substitute (93) into the LHS of (91), and solve for the value of a^*_{1x}.

$$a^*_{1x} = \frac{1}{4}\left(14 + a_{1x} + \sqrt{356 - 28a_{1x} + {a_{1x}}^2} - \sqrt{2}\sqrt{596 + {a_{1x}}^2 + (14 + a_{1x})\sqrt{356 - 28a_{1x} + {a_{1x}}^2}}\right) \quad (95)$$

$$a^*_{1x} = \frac{1}{2}(7 - \sqrt{129}) \quad (96)$$

I then try the positive root. I substitute (94) into the LHS of (92) and solve for the value of a_{1x}^*.

$$a_{1x}^* = \frac{1}{4}\left(14 + a_{1x} + \sqrt{356 - 28a_{1x} + {a_{1x}}^2} + \sqrt{2}\sqrt{596 + {a_{1x}}^2 - (14 + a_{1x})\sqrt{356 - 28a_{1x} + {a_{1x}}^2}}\right) \tag{97}$$

$$a_{1x}^* = \frac{1}{2}(7 + \sqrt{129}) \tag{98}$$

The optimal value for a_{1x}^* has a negative and positive solution. However, knowing that $40\ln(a_{1x} + a_{1y} - 14)$ is a component of the welfare function, the sum of a_{1x} and a_{1y} must be greater than 14, otherwise the function is undefined. a_{1x}^* in (96) is a negative number, so it cannot be a valid solution for a_{1x}. Therefore, $a_{1x}^* = \frac{1}{2}(7 + \sqrt{129})$ is the solution. Since we know that the game is symmetrical and $a_{1x}^* = a_{1y}^*$:

$$a_{1x}^* = a_{1y}^* = \frac{1}{2}(7 + \sqrt{129}) \tag{99}$$

The next step is substitute the decision profile (a_{1x}^*, a_{1y}^*) back into W_T.

$$\begin{aligned} W_T = &-\frac{1}{4}(7 + \sqrt{129})^2 - i_x - i_y + 40\ln[-7 + \sqrt{129}] \\ &+ \frac{9}{10}\left(-\frac{\left(14 + 7i_x + \sqrt{(2 + i_x)^2(129 + 20i_x + 20i_y)}\right)^2}{(2 + i_x)(4 + i_x + i_y)^2} - \frac{\left(14 + 7i_y + \sqrt{(2 + i_y)^2(129 + 20i_x + 20i_y)}\right)^2}{(2 + i_y)(4 + i_x + i_y)^2}\right) \\ &+ 36\ln\left[\frac{-28 - 7i_x - 7i_y + \sqrt{(2 + i_x)^2(129 + 20i_x + 20i_y)} + \sqrt{(2 + i_y)^2(129 + 20i_x + 20i_y)}}{4 + i_x + i_y}\right] \end{aligned} \tag{100}$$

Now that we have W_T in terms of just i_x and i_y, I find the BR functions of i_x and i_y by taking the partial derivative of W_T with respect to i_x and i_y. The terms for $\frac{\partial W_T}{\partial i_x}$ and $\frac{\partial W_T}{\partial i_y}$ were even more unwieldy compared to $\frac{\partial W_x}{\partial i_x}$ and $\frac{\partial W_y}{\partial i_y}$. Even in its simplified form, it would take up three quarters of a page to write out each expression. Therefore, I will skip straight to plotting their level curves out onto a graph (Figure 9):

Figure 9, shows two BR functions in the graph, perfectly overlapping. I verified this by plotting the BR functions separately as well. Any point along this curve is a solution that maximizes W_T. This means there are an infinite number of combinations of i_x and i_y that maximize the total welfare function. Assuming the single-decision-maker wants the decision profile that divides welfare evenly between both countries, then we need to apply the condition $(i_x^* = i_y^*)$. After making this condition, we get the solution (13.64, 13.64) for (i_x^*, i_y^*). I substitute these numbers back into the decision profile (a_{2x}^*, a_{2y}^*) to get optimal abatement levels in period 2. Now that I have solved for all the decision variables, I can substitute them back into W_T. These are the solutions of the optimal model under a scenario with no technology spillovers:

$$i_x = i_y = 13.64$$
$$a_{1x} = a_{1y} = 9.18$$

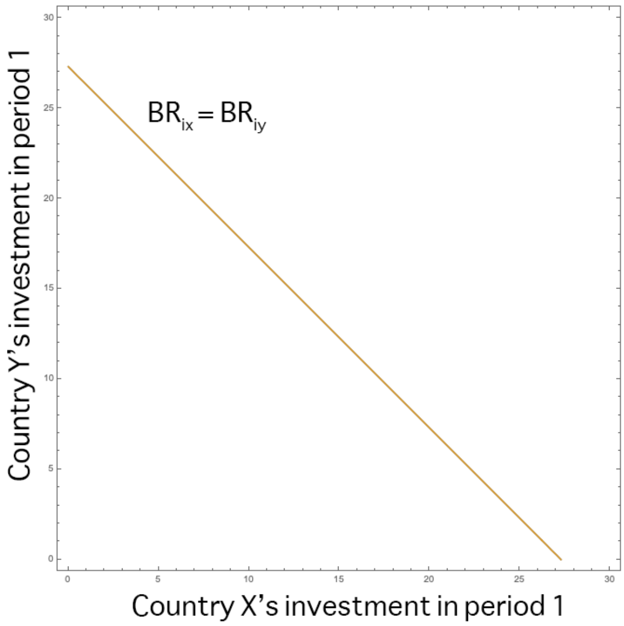

Figure 9: Graphing the BR functions for i_x and i_y to find the strategy profile (i_x^*, i_y^*). Figure 9 is the same as Figure 5 from subsection 4.1.b.

$$a_{1x} = a_{1y} = 16.5$$

$$W_T = 44.03$$

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