Economics Honors Thesis, Spring 2021

# Dynamic Incentives and Effort Provision in Professional Tennis Tournaments


**Department of Economics**
**University of California, Berkeley**

Author:

Ruiwen Pan

Advisor:

Dr. Benjamin Handel


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**Abstract**

In this paper, I study high-stakes professional tennis elimination tournaments and examine the impact of various forward-looking incentives and historical player-to-player differentials on effort exertion and match outcomes. In particular, drawing on theoretical foundations from Tournament Theory, I use statistical models to study the impact of extrinsic monetary incentives on the probability that the better-ranked player wins a tennis match. I find significant results that suggest steeper tournament prize structures significantly increase the likelihood of better-ranked players winning a match, even after controlling for relative player abilities, historical match-ups, and prior tournament effort. Lastly, I discuss a dynamic model for further research that frames the tennis season as a constrained optimization problem whereby players choose a level of effort, each with its associated costs, to maximize prize money subject to a finite budget constraint of physical effort.



**Keywords:** *Elimination Tournaments, Tournament Theory, Prize Spread, Effort Choice, Dynamic Contest, Effort Spillover, Sports Analytics*

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# 1 Introduction

One of the most everlasting puzzles in any professional sports setting is the possibility that a dark horse or underdog competitor pulls off an extraordinary and unexpected upset of a prominent star player or "favorite". There have been many rationales put forward by close followers of sports, notably tennis, on why this occurs (roughly 30% of the matches). Can underlying skill differentials between players explain this, or do monetary incentives play a role too? Is it empirically observable that upsets occur more frequently in high-stakes tournaments than lower prestige events? Does fatigue as a function of previous exerted effort drive up the possibility of an upset? Lastly, what if upsets are just a result of "luck" and there is no systematic structure to this phenomenon?

In this paper, I use advanced econometric techniques on tennis data to study the probability of the better-ranked (stronger) player winning a match while controlling for a wide variety of player differentiating factors. Then, I isolate and estimate the impact of round-to-round monetary incentives and test whether a steeper prize structure overall generates more benefits for the stronger player. I will also examine whether tennis tournaments are effective at "selecting the best" throughout the sequential rounds of the tournament, as theory would argue is the purpose of tournament design. In addition, I estimate the difference in the likelihood of the stronger player winning between Grand Slam and Masters 1000 tournaments. After making appropriate adjustments for tournament format, I ask the following question: how much more on average does the tournament need to pay the stronger player in a less prestigious Masters tournament to make them try as hard as in a more prestigious Grand Slam event?

The effects of prize structure from this paper are also of great relevance with current debates within the international tennis community regarding inequality in tennis tournament prizes and, most notably, the prizes for early-round losers. In recent years, many prestigious Grand Slam tournaments have been increasing the consolation prize money for first-round losers to help these players adjust and make ends meet. Tennis is an expensive sport that requires money for travel, equipment, and coaching. These are burdensome expenses for sponsorless younger and middle-tier players who have yet to make a name for themselves. The results from this paper can help answer whether a higher safety net in the form of early tournament prizes incentivizes would bolster the feasibility of the careers of young tennis players and their prospects of making more successful runs in prestigious tournaments.

Lastly, near the end of the paper, I briefly mention extensions to this paper for further research and propose a simple model of the decision-making process of tennis players making decisions to conserve and allocate their effort over tournaments in the tennis season.

## 2 Literature Review

Tournament designers want to design a competitive contest, with the goal of "selecting the best" in mind, that picks out the best candidate among a pool of heterogeneous candidates. As a result, in the contest literature, tournaments often serve as a selection mechanism by which the relative abilities of all candidates are unveiled and determined, often through multi-stage competition and elimination. In addition, tournaments often have prizes associated with them, and the stakes of these prizes vary by each subsequent stage of the contest. As such, contests like these are also incentive mechanisms that seek to bring about and motivate greater effort exertion on the part of contest participants. Some prominent economists have studied, and developed theory on this front, the most notable of which is a branch of personnel economics called Tournament Theory.

Tournament Theory, first developed by Edward Lazear and Sherwin Rosen, is often used to analyze contests and competitions where wage differences between workers are based upon relative performance differences between these individuals, not absolute marginal productivity. One of the first papers that laid out the groundwork for what became Tournament Theory is Lazear and Rosen's paper in 1981 entitled *Rank-Order Tournaments as Optimum Labor Contracts*. In their study of performance-related pay, Lazear and Rosen (1981) compared the piece-rate conventional payment system with a provocatively different compensation scheme based solely on the ordinal rank of workers in an organization, which are then called rank-order tournaments. The benefit of this type of compensation scheme shines when individual productivity is difficult to measure using a cardinal scale, which is especially likely when the signal-to-noise ratio is low, and assessments of individual value-added are contaminated. The authors argued that workers within an organization could be rewarded based upon their rank while at the same time allowing the organization to reach the same efficient allocation of resources as a compensation structure based on individual output levels.

Rosen (1986) then followed up upon his recent work from 1981 and studied the prospect of sequential elimination tournaments instead of rank-order tournaments. According to Rosen, the purpose of elimination tournaments is to promote the "survival of the fittest", and rewards are needed to maintain the quality of play in later stages of the tournament. Furthermore, it follows that the reward structure in the tournament influences the quality and competitiveness of each stage of the tournament. The performance incentive at each stage of the tournament is the difference between the winner's prize and the loser's prize, of which the former is uncertain and the latter is guaranteed. Winning the current match unlocks access to future advancement opportunities, which is similar to climbing successive rungs on the career ladder. In the last round, advancement opportunities vanish, and the final stage has a “no-tomorrow” aspect. As a result, Rosen argues that a kink in the form of a steep increase in prize money is necessary for the final stage of the tournaments. The steep increase is to essentially extend the horizon for high-skilled contestants, which motivates continued effort by giving them an impression that there are more rungs in the ladder to climb, regardless of how far the contestant has climbed prior. The kink implies that the structure of prizes at the top has profound ripple effects further down in the ladder, which directly impacts players’ earlier benefit-cost analyses of effort provision decisions. These decisions consist of assessments of one’s own expended effort, future competition, and the prospect of survival, both forward-looking and recursive, of all players in any match.

Tournament Theory has many practical applications that can help rationalize why a giant pay spike follows a promotion from a Vice President to a President in a firm. It is doubtful that the marginal productivity of the individual has doubled or tripled after the promotion. In short, this handsome salary increase is the reward offered to those in the organization who made enough effort to climb the ladder to attain this coveted position level. A central proposition that this theory posits is that the prize spread needs to be continually higher in each subsequent round to continue to incentivize the same amount of effort exertion. Each subsequent round will come with higher stakes, which will induce continual effort on the part of the contestants. The incentive to put in more effort increases as a function of the prize spread, which is the difference between the winning and losing prizes. The fact that the spread drives contestants’ effort levels, which is a type of reference point dependence, is very elegant considering it is also a feature exhibited in prospect theory from behavioral finance. As a result, firms have the incentive to increase the prize spread and find an optimal spread level that is high enough to induce worker investment but low enough that this investment is not too costly for the worker. There are many beneficial properties of a tournament structure from an organization’s point of view, as tournament and prizes can motivate worker effort and performance, select the best workers, match worker with the job, and act as an incentive to encourage lower turnover at the organization.

Tournament Theory is frequently applied in professional athletics. Individual sporting events fulfill many of the criteria that the theory requires, such as clear measures of the relative abilities of players, relative ease in observing the connection between effort and performance, and incentives that are determined ex-ante. Brown (2011) argued that the relative abilities of workers highly impact benefits of internal competition on worker effort, and she uses PGA Tour data to test her hypothesis. She finds that a superstar, like Tiger Woods in golf, leads to reduced performance from competitors. Additionally, this so-called “Superstar Effect”, the adverse effect of a superstar on their competitors’ performance levels, is of higher magnitude for higher-skilled golfers than lower-skilled ones. In addition, she demonstrates robustness in her results by showing that the reduced performance stemming from the Superstar Effect does not come from media attention nor greater risk-taking in play style. The adverse superstar effect only arises when Woods' performance is superstar level and not when his performance is waning. As a result, she demonstrates that high heterogeneity between the relative skills of contestants may, on aggregate, reduce effort.

Tennis tournaments in particular have also been the subject of study by economists. Brown and Minor (2014) studied how past, current, and future competition within an elimination tournament affect the probability that the stronger player wins. They present a model and use tennis data to test the presence of a "shadow effect" and an "effort spillover effect." The authors' first proposition, the shadow of future competitors, is that the prospect of facing a strong expected future competitor in a future match will act as a shadow in the present and decrease the probability that the stronger player wins in the current match. The second main proposition, effort spillover between stages, states that effort from previous matches of the tournament will "spillover" into the current match and will decrease the probability that the stronger player wins in the current match. As an aside in their paper, the authors claim that their theoretical model also predicts that a steeper prize structure improves the stronger player's probability of success in all stages, which is a topic this paper will explore in detail as well.

# 3 Data

The Association of Tennis Professionals, known as the ATP, is formed in 1972, and it is the official organizer of the men's circuit of professional tennis tournaments around the world. In 2019, the ATP World Tour featured 63 tournaments of varying levels taking place in 31 countries across six continents in addition to the four "Grand Slams". These tour-level continents range in size from 32 to 128 and differ in many respects, ranging from total purse (prize money) and the number of rankings points given out, directly impacting the ATP World Ranking of a player. Higher ranked players automatically qualify for prestigious tournament events and sometimes paid an appearance fee for lower-profile events. Furthermore, highly ranked players are also often given a seed in the initial tournament draw based on the current ranking. In contrast, lower-ranking players are unseeded and often need to succeed in qualifiers or earn a wild card to enter a higher-profile tournament. These lower-ranked players will likely play against a seeded high-ranked player in the first round.

Professional Tennis tournaments satisfy many prerequisites to qualify as an environment to test predictions from Tournament Theory. Tennis tournaments are multi-round and single elimination tournaments where only winning players can advance to subsequent rounds while losing players are eliminated. The prize money associated with each stage increases further into the tournament, and losers also receive a losing prize associated with the round in which they were eliminated. Furthermore, the prize money for each tournament is known ahead of time before the start of each tournament.

In addition, in professional tennis data, up-to-date implied underlying abilities of all participating players can easily be observed in the tournament from their ATP World Tour Ranking. As tournament draws are announced before the tournament's start, it is with great ease for all players to anticipate, and sometimes even be confident of, future match-ups in the next rounds of the tournament and calculate paths to the championship.

Due to the ease of observing up-to-date player heterogeneity and prize money information, I can use these readily available data to study the relationship between monetary incentive and match outcomes and control for relative skills, history, and other differentiating factors between players.

## 3.1 ATP Match Data

In order to set out to test the predictions from Tournament Theory and properly use econometrics to evaluate the probability of the stronger player winning in a match, I use real tennis match-level data between 2011 to 2019 in ATP Masters 1000 and Grand Slam events. This data set is available from Jeff Sackmann, whose Github repository contains all historical tennis match data since 1968. The data set on the repository contains detailed match-level information and statistics about the winning and losing players of each match, such as height, weight, country of origin, and ranking at the time of the match. Each observation in the data set also contains statistics related to the match, such as minutes played, the round of the match, tournament name, first serves made, breakpoints faced, and many more.

The analysis in this paper only focuses on matches between 2011 to 2019, and unfinished matches due to players retiring mid-match are filtered out. Many variables used for analysis are not found in the original data set, and I used Python data structures and the pandas library to create them from the existing data. One example of a created variable includes the Head-to-Head (H2H) historical record of past head-to-head matchups between the two players before their current match. Next, I added a binary variable for whether the better-ranked (stronger) player wins a given match. In addition, since the data in terms of winning and losing players, I adjusted the data to have relevant variables point toward the stronger and weaker players, respectively.

This paper will focus on the “Masters 1000” and “Grand Slam” events. These are the top two premier tennis events recognized by the Association of Tennis Professionals (ATP). The Grand Slam events are the most prestigious tennis events out there and offer the most lucrative prize money and attract all the best players. The four Grand Slams are the Australian Open, French Open (Roland Garros), Wimbledon, and the US Open. These offer maximum ATP rankings points of 2000 for the winner and are composed of 7 rounds with 128 participating players. The second most prestigious tier of tennis tournaments is ATP Masters 1000 events, more commonly known as Masters Events. Most Masters events, by contrast, are 6-round tournaments except for the Indian Wells Masters and the Miami Open, which are two events held back-to-back in the spring of the year.

### 3.1.1 Description of Variables

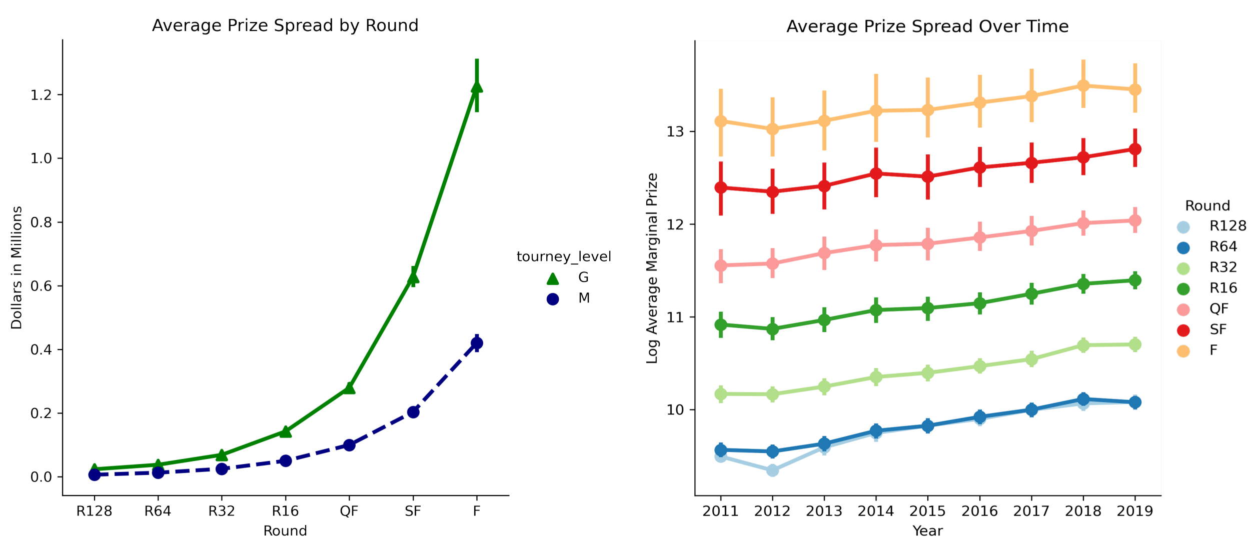
The Prize Spread is defined as $\Delta Prize = Prize_{r+1} - Prize_r$, which represents the difference between the current round’s winning prize (next round’s prize) and losing prize (current round’s prize). In the finals of a tournament, the prize spread is the difference between the champion’s prize and the runner-up’s prize. The Marginal Points variable, known as the points spread, is defined the same way as the prize spread, but in terms of ATP rankings points. The Grand Slam variable is a binary variable representing whether a given match is in a Grand Slam tournament. The Strong Wins variable is an indicator representing whether the better-ranked player wins in the current match. The Het variable represents player absolute heterogeneity, which is the absolute value of the difference in ranking of the two players in a given match. Similarly, the Current Rank Ratio is another measure of player heterogeneity and is defined as the ratio of the worse-ranked to the better-ranked player. Strong H2H Wins and Weak H2H Wins are defined as the head-to-head wins won by both the stronger and weaker players in their previous head-to-head match-ups. Strong Games Played and Weak Games Played are the number of games played in previous rounds of each tournament. The initial values of these two variables are set to zero for the very first round of every tournament. Lastly, Strong Exp and Weak Exp are the years of experience since the professional debut of the stronger and weaker players. I decided to increase the precision of this variable to include the month and day using the fractional year format. For example, 2019-07-01 will be 2019.495890 in fractional year format. The variable for experience is simply the arithmetic difference between the fractional date of the match and the date of the player’s debut.

Table 1 presents summary statistics for the main variables that are to be used in the analysis to come in the results section.

Table 1: Table of Summary Statistics

| Statistic | N | Mean | St. Dev. | Min | Max |
|---|---|---|---|---|---|
| Prize Spread | 9,078 | 60,276.010 | 128,460.700 | 4,850 | 1,950,000 |
| Log Prize Spread | 9,078 | 10.346 | 0.968 | 8.487 | 14.483 |
| Marginal Points | 9,078 | 69.759 | 88.537 | 15 | 800 |
| Grand Slam | 9,078 | 0.480 | 0.500 | 0 | 1 |
| Strong Wins | 9,078 | 0.702 | 0.457 | 0 | 1 |
| Het | 9,078 | 56.554 | 81.734 | 1 | 1,383 |
| Current Rank Ratio | 9,078 | 7.753 | 18.139 | 1.009 | 570.000 |
| Strong H2H Wins | 9,078 | 1.550 | 2.835 | 0 | 28 |
| Weak H2H Wins | 9,078 | 0.751 | 1.855 | 0 | 26 |
| Strong Games Played | 9,078 | 21.708 | 33.415 | 0 | 288 |
| Weak Games Played | 9,078 | 27.633 | 36.706 | 0 | 244 |
| Strong Exp | 9,078 | 8.298 | 3.936 | 0.000 | 19.721 |
| Weak Exp | 9,078 | 6.659 | 4.254 | 0.000 | 19.452 |

*Notes:* Data contains ATP Masters 1000 and Grand Slam Events from 2011 to 2019. Unfinished matches, also known as retired matches, are excluded.




((a)) Steepness of Prize Structure

((b)) Growth of Prize Spread

Figure 1: Average Prize Spread by Round, Year, and Type

## 3.2 ATP Prize Money Data

In order to connect round-to-round prize spread information with match-level information from the ATP Matches data, data related to tournament-specific purse and round prize information is needed. To this end, I extract the purse information from all Masters and Grand Slam tournaments between 2011 and 2019 from perfect-tennis.com. This website contains information for all ATP tournaments' prize money by round and year and lists them in multiple currencies, using exchange rates on the date of the final of that year's tournament. Additionally, this website provides information such as the annual percent change of each round's prize money. For the rest of the paper, tournament prize information is adjusted for inflation and listed in 2011 US dollars.

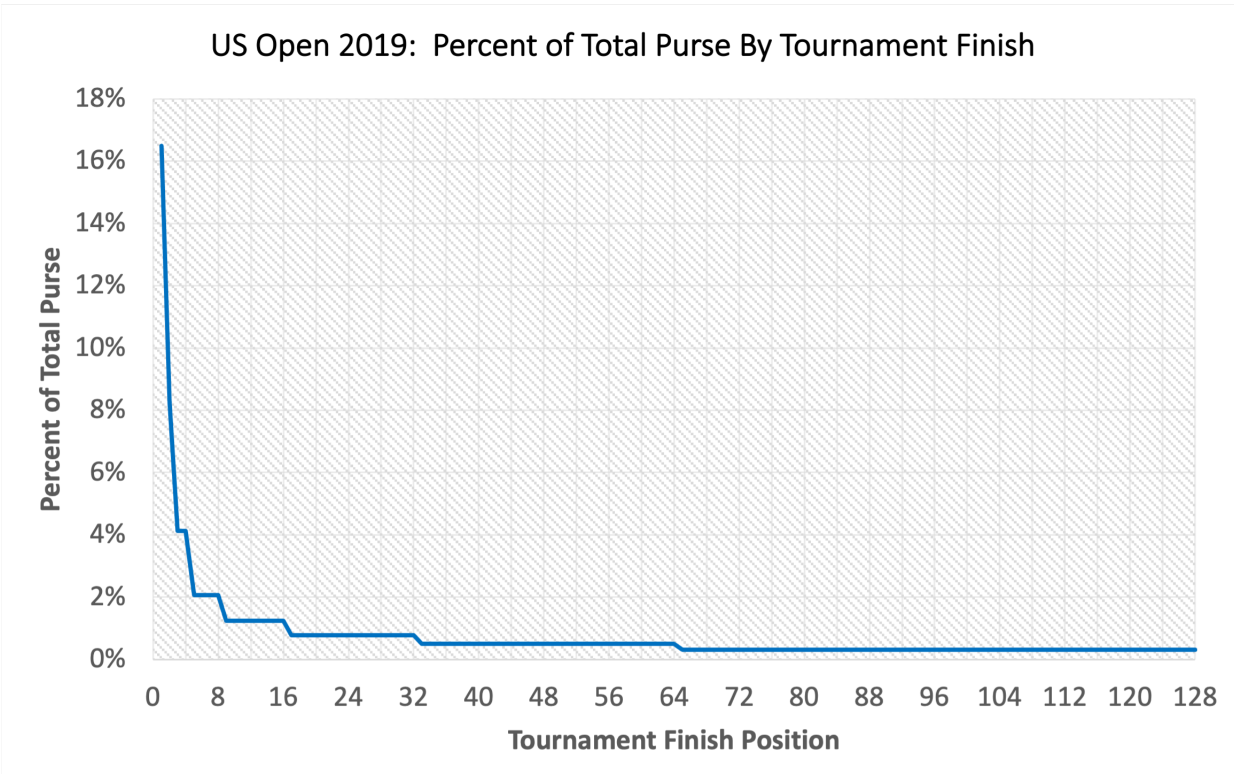



Figure 2: Distribution of Prizes in US Open 2019

Figure 1 and Figure 2 present exploratory data analysis and visualizations of the steepness of the prize structure, growth of prize spread over time, and the distribution of prize money in tennis tournaments. As we can see, in panel (a) of Figure 1, the prize spread of Grand Slam tournaments are much steeper than for Masters events and, for both events, the next round's prize is roughly a multiple of 2 compared to the current round's prize. Instead, if we plot the same graph using a log axis, then the two lines are linear. In panel (b) in Figure

1, we can see that the growth of log average prize spread is roughly the same shape for all rounds from 2011 to 2019. However, in absolute terms, the higher-stakes round's prize spreads have increased much more than lower rounds. Lastly, Figure 2 shows the percentage of total purse won based on tournament finish for the 2019 US Open. This distribution structure is very typical across virtually all tournaments: the winner takes a large fraction of the total purse, and the top four players take home roughly 33% of all prize money.

# 4 Results

This section presents empirical econometric models to test whether the prize spread induces more significant benefitS for the stronger player. Then, I use the results from the model to study differences between Grand Slam and Masters tournaments, also known as the "Slams-Masters gap". Lastly, I conduct a robustness check on the differences to see whether the gap is driven by a change in the tournament's play format.

## 4.1 Prize Spread on Match Outcomes

With the following logistic regression specification, I estimate the effect of the prize spread on the probability of the stronger player winning, controlling for player heterogeneity, rankings incentives, previous match effort, and previous history:

$$ln\Big(\frac{Pr(StrongWins_{m,r,t})}{1-Pr(StrongWins_{m,r,t})}\Big) = \beta_0 + \beta_1 ln(PrizeSpread_{m,r,t}) + \beta_2 CurrentRankRatio_{m,r,t}$$

$$+\beta_3 StrongH2H_{m,r,t} + \beta_4 WeakH2H_{m,r,t} + \beta_5 GamesPlayedGap_{m,r,t} + \beta_6 Points_{m,r,t}$$

$$+\beta_7 ExperienceGap_{m,r,t} + \beta_8 GrandSlam_{m,r,t} + \sum_{e=1}^{E} \gamma_e Event_t + \epsilon_{m,r,t}$$

As before, $StrongWins_{m,r,t}$ is a binary variable of whether the better-ranked player won in match $m$, of round $r$, in tournament $t$. The term inside the log on the left-hand side is known as the "odds ratio", which is the ratio of the probability of the stronger player winning relative to the probability of the weaker player winning. $PrizeSpread_{m,r,t}$ is the prize spread, adjusted for inflation, between the winner's prize and the loser's prize; $CurrentRankRatio_{m,r,t}$ is the ratio of the weaker and stronger player's ranks; $StrongH2H_{m,r,t}$ and $WeakH2H_{m,r,t}$ are the number of head-to-head matches won by the stronger and the weaker player, respectively; $GamesPlayedGap_{m,r,t}$ represents the difference in the number of games played by the stronger player and the weaker player in previous rounds of the tournament; $Points_{m,r,t}$ is the spread between the winner and loser's reward of ATP ranking points; $ExperienceGap_{m,r,t}$ is the difference between the stronger and weaker player's experiences in years; $Event_t$ is a full set of event-specific dummy variables (e.g. 2019 US Open) representing "tournament-year" fixed effects. These event-level fixed effects absorb average event-specific variation, including but not limited to media attention, weather, player momentum, both observed and unobserved. Lastly, $\epsilon_{m,r,t}$ is the error term.

Table 2 presents the results of several logistic regression specifications with results listed in the odds ratio format. Like Brown and Minor (2014), all regression models are run with heteroskedasticity-robust standard errors clustered at the tournament-year (event) level to account for correlations and momentum in player performance during a given tournament event.

### 4.1.1 Results from Logistic Regression

Table 2 below shows all of the odds ratios, with associated standard errors, for each of the five regression specifications. The odds ratios on *PrizeSpread* for all of the pooled regressions are all above 1, meaning that the stronger player is more likely to win, holding all else constant. Furthermore, these estimates are all statistically significant. For instance, in regression (2), where *Het* is the proxy for player heterogeneity, every one log point increase in the prize spread, on average, leads to the odds of the stronger player winning increasing by a factor of 1.4, holding all else constant. On the other hand, in regression (3), where the *CurrentRankRatio* is used as the measure of heterogeneity, the stronger player is 1.15 times more likely to win on average than the weaker player for every one log point increase in prize spread, ceteris paribus. For the rest of the paper, regression (3) will be used in upcoming analyses, as the estimate proves to be more cautious and reliable. If the regression is run on the Grand Slam events only, as in regression (4), the coefficient on log prize spread is not significantly different from 1. However, for Masters tournaments, the odds ratio is roughly 1.3 and statistically significant. Upon further consideration, the intuition of the result could be as follows: since the honor of winning a Grand Slam is much more prestigious, and these tournaments attract players to work hard for many other reasons than prize money (records, reputation), highly-ranked players would exert more effort and compete more fiercely even in the very beginning of the tournament. Additionally, with the play format of Grand Slams being best out of five instead of best out of three, it becomes an even more daunting task for worse-ranked players to pull an upset of a better-ranked player.

Table 2: Logistic Regression Odds Ratios Results

| | Stronger Player Wins in Current Match (0 or 1) | | | | |
|---|---|---|---|---|---|
| | Pooled | Pooled | Pooled | Slams | Masters |
| | (1) | (2) | (3) | (4) | (5) |
| Constant | 0.275*** | 0.042*** | 0.302* | 1.497 | 0.156** |
| | (0.128) | (0.028) | (0.201) | (1.553) | (0.135) |
| Log Prize Spread | 1.184*** | 1.408*** | 1.151** | 1.009 | 1.304*** |
| | (0.058) | (0.085) | (0.070) | (0.109) | (0.122) |
| Het | 1.004*** | 1.005*** | | | |
| | (0.001) | (0.001) | | | |
| Current Rank Ratio | | | 1.074*** | 1.106*** | 1.050*** |
| | | | (0.011) | (0.019) | (0.012) |
| Strong H2H Wins | 1.126*** | 1.115*** | 1.077*** | 1.061*** | 1.088*** |
| | (0.016) | (0.016) | (0.015) | (0.024) | (0.019) |
| Weak H2H Wins | 0.873*** | 0.869*** | 0.894*** | 0.921*** | 0.874*** |
| | (0.015) | (0.015) | (0.015) | (0.027) | (0.019) |
| Games Played Gap | 0.988*** | 0.988*** | 0.992*** | 0.990*** | 0.994** |
| | (0.002) | (0.002) | (0.002) | (0.002) | (0.003) |
| Experience Gap | 1.022*** | 1.021*** | 1.014*** | 1.021*** | 1.007 |
| | (0.005) | (0.005) | (0.005) | (0.007) | (0.006) |
| Marginal Points | 0.999* | 0.998*** | 0.999* | 1.000 | 0.998* |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| Grand Slam | 1.261*** | 1.802 | 1.614 | | |
| | (0.071) | (0.671) | (0.591) | | |
| Fixed Effects | No | Yes | Yes | Yes | Yes |
| Observations | 9,078 | 9,078 | 9,078 | 4,356 | 4,722 |
| Log Likelihood | −5,303.986 | −5,209.932 | −5,112.957 | −2,293.220 | −2,803.923 |
| Akaike Inf. Crit. | 10,625.970 | 10,663.860 | 10,469.910 | 4,672.440 | 5,779.846 |

*Note:* *p<0.1; **p<0.05; ***p<0.01
Odds ratios and their associated standard errors are shown.

#### 4.1.2 Empirical Differences Between Grand Slams and Masters

Figure 3 shows the fitted values, or predictions, of the pooled logistic regression model (Model 3) on the complete data. The fitted values from the model are then grouped by round and plotted as lines. As shown in Figure 3 below, there is a clear gap between the Grand Slams and the Masters events, the magnitude of which varies significantly by round.

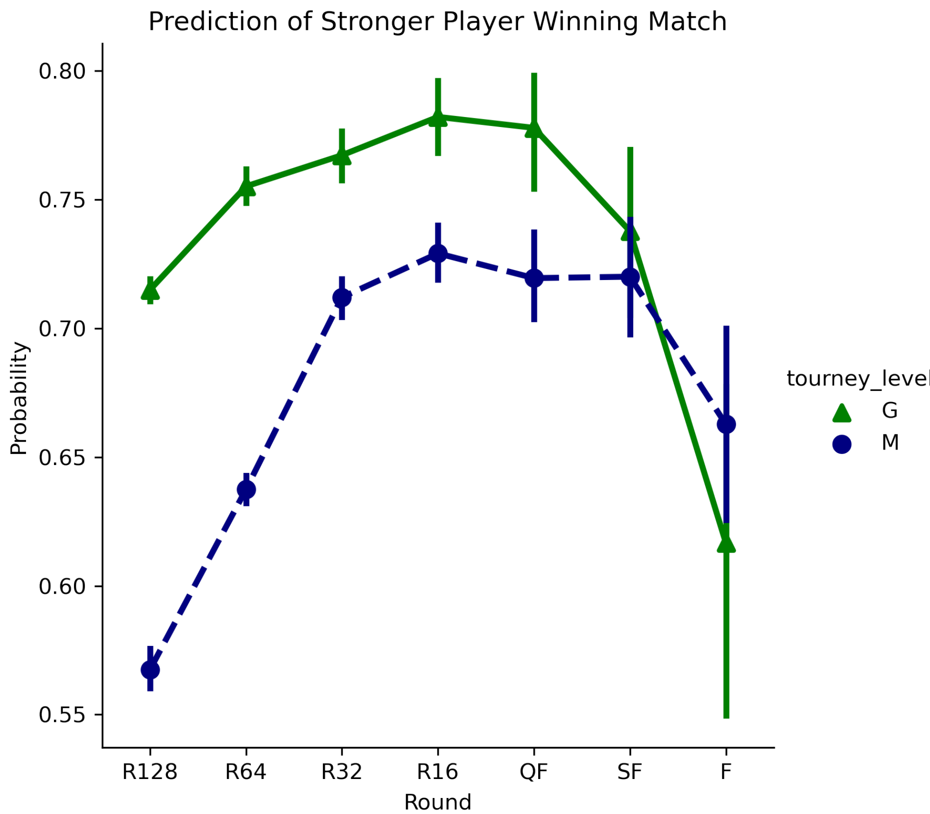



Figure 3: Prediction of Stronger Player Winning Match by Tournament Level

The “Slams-Masters Gap” is largest in the first round of the tournament, and both lines reach their peaks in R16, the fourth round, and then tapers off and drops down. Before the final round, the probability of the stronger player winning is always higher than that for the Masters. In terms of theory, it appears that the selection power of tournaments using the prize spread is increasing in effectiveness up to R16, where it hits the top. After that point, the selection capability of tournaments becomes less effective as the stakes get higher, likely due to interference from other psychological effects such as choking or the fact that the degree of absolute heterogeneity between players goes down further into the tournament as the competition increases. Additionally, fatigue resulting from costly effort from previous rounds becomes more apparent further into the tournament. According to

Brown and Minor (2014), the effort spillover from previous stages has a more pronounced effect on the stronger player, which could be why we observe the decline in probability after halfway of the tournament. Lastly, as with Brown and Minor's work, the shadow of future competition, which adversely affects the stronger player, is growing in strength as the final approaches, likely due to psychological pressure of high stakes, and causes a decline of probability in the probability of stronger players winning. Noticeably, this drop in probability is substantial in the finals. In the finals, the Slams-Masters gap reverses, and the probability of an upset becomes more likely in a Grand Slam. Possible explanations for this reversal include the powerful "choking under pressure" effect that affects the better-ranked player more strongly than the worse-ranked player and is more pronounced for Grand Slams than Masters events.

## 4.2 Robustness Check

The two lines that make up the Slams-Masters gap shown in Figure 3 above may not be directly comparable since the two types of events differ by another crucial factor besides the prestige and the monetary reward. Grand Slams play a best-of-5 sets tournament format while the Masters tournaments only play best-of-3 sets. To adjust for this difference, I estimate the counterfactual probability of winning a best-of-5 match as if it were played at a Masters event. This adjustment would adjust for the differing play formats and make Grand Slams and Masters tournaments more comparable. The following paragraphs describe the process to conduct the counterfactual simulation.

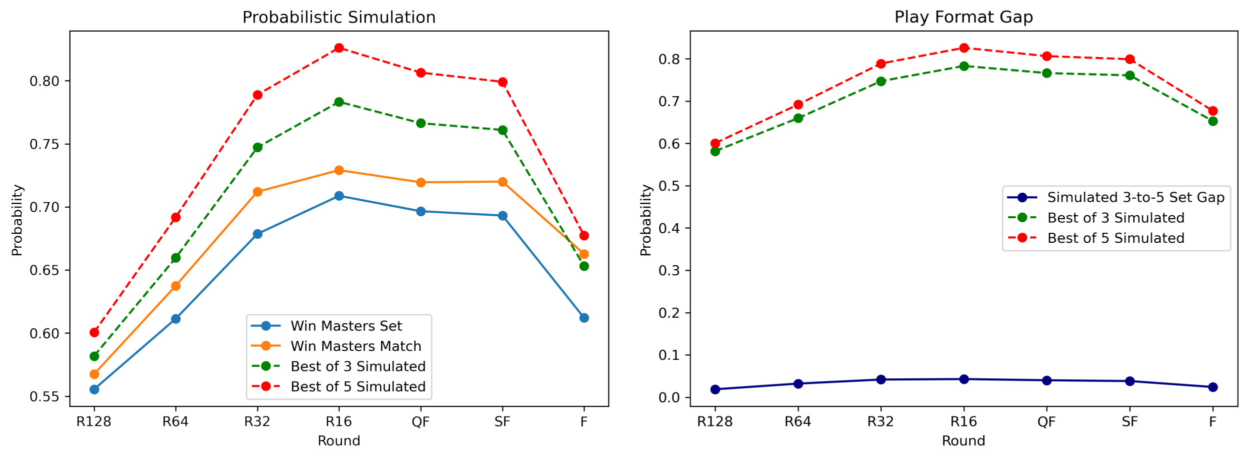
### 4.2.1 Counterfactual Simulation Exercise

Let $p$ be the probability of winning a set for the stronger player. Assume for simplicity that each of the sets in the match is independently and identically distributed (iid). Then, if we have the stronger player's probability of winning a set, then we can calculate his probability of winning a match using combinatorics and probability theory:

$$Pr(\text{Win a Best-of-3 Match}) = p^2 + 2p^2(1-p) \tag{1}$$

$$Pr(\text{Win a Best-of-5 Match}) = p^3 + 3p^3(1-p) + 6p^3(1-p)^2 \tag{2}$$

In order to calculate $p$, the probability of winning a set, I estimate the following regression on the Masters tournaments only, where $StrongSetWonProportion \in \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ represents the proportion of sets won by the stronger player in best-of-3 matches. The probability of




((a)) Simulated Counterfactuals ((b)) 3-to-5 Set Gap

Figure 4: Results from Simulation

the stronger player winning any given set can be predicted with the following OLS regression specification:

$$StrongSetWonProportion_{m,r,t} = \beta_0 + \beta_1 ln(PrizeSpread_{m,r,t}) + \beta_2 CurrentRankRatio_{m,r,t} + \beta_3 StrongH2H_{m,r,t} + \beta_4 WeakH2H_{m,r,t} + \beta_5 GamesPlayedGap_{m,r,t} + \beta_6 Points_{m,r,t} + \beta_7 ExperienceGap_{m,r,t} + \sum_{j=1}^{J} \gamma_j MastersEvent_t + \epsilon_{m,r,t}$$

Results from the above regression are shown in Table 3. The following OLS regression fitted values represent $p$ from equations (1) and (2) above. Then, I use the fitted values of the new regression and plot them as the "Win Masters Set" line in Figure 4. Equations (1) and (2) are applied to all Masters matches to simulate the inferred probability of winning a best-of-3 match and the counterfactual probability of winning a best-of-5 match, assuming all independence assumptions hold. The "Best of 5 Simulated" line is the counterfactual probability of winning a best-of-5 match at a Masters event, aggregated by round. The difference between the two simulated lines is the "3-to-5 Set Gap". This gap quantifies the increase in the likelihood of the stronger player winning (counterfactual) best-of-five matches, relative to best-of-three matches, at the same Masters event. It should also be noted that the "Best of 3 Simulated" line and the "Win Masters Match" line are not the same, as the independence assumption does not hold up due to the existence of momentum between sets. The lines in the two panels in Figure 4 show the simulation counterfactual exercise visualization.

Table 3: OLS Regression Robustness Check Results

| | Proportion of Sets Won By Stronger Player | |
|---|---|---|
| | Masters | Slams |
| | (1) | (2) |
| Constant | −0.005 | 0.464*** |
| | (0.148) | (0.142) |
| Log Prize Spread | 0.071*** | 0.020 |
| | (0.016) | (0.015) |
| Current Rank Ratio | 0.003*** | 0.003*** |
| | (0.001) | (0.0004) |
| Strong H2H Wins | 0.011*** | 0.015*** |
| | (0.002) | (0.002) |
| Weak H2H Wins | −0.022*** | −0.019*** |
| | (0.004) | (0.004) |
| Games Played Gap | 0.002** | 0.005*** |
| | (0.001) | (0.001) |
| Experience Gap | −0.002*** | −0.002*** |
| | (0.0004) | (0.0003) |
| Marginal Points | −0.001*** | −0.0003** |
| | (0.0002) | (0.0001) |
| Tournament-Year Fixed Effects | Yes | Yes |
| Observations | 4,722 | 4,355 |
| $R^2$ | 0.072 | 0.079 |
| Adjusted $R^2$ | 0.055 | 0.070 |
| Residual Std. Error | 0.376 (df = 4636) | 0.320 (df = 4312) |
| F Statistic | 4.213*** (df = 85; 4636) | 8.850*** (df = 42; 4312) |

*Note:* Coefficients and associated standard errors shown.

#### 4.2.2 Adjusted Slams-Masters Gap

Now that the impact of the change in play format between Grand Slams and Masters has been isolated and estimated, I will adjust the Slams-Masters gap first shown in Figure 3 accordingly. Since the intent of the counterfactual simulation exercise is to infer the probability of the stronger player winning hypothetical best-of-5 match at a Masters event, I can simply add the "Simulated 3-to-5 Set Gap" line from panel (b) of Figure 4 to the original Masters prediction line in Figure 3. After the adjustment, the difference between the original Slams-Masters gap diminished overall, with especially reduced differences in the mid-tournament rounds. Figure 6 shows the newly adjusted Slams-Masters predictions, with the difference between the blue and dotted green line being called the "adjusted gap".

| | Unadjusted Gap | | | | Adjusted Gap | | | |
|---|---|---|---|---|---|---|---|---|
| Round | T-statistic | Slams Mean | Masters Mean | Difference | T-statistic | Slams Mean | Masters Mean | Difference |
| **R128** | 33.1611 | 0.7150 | 0.5675 | 0.1475 | 24.7673 | 0.7150 | 0.5863 | 0.1287 |
| **R64** | 27.2305 | 0.7552 | 0.6375 | 0.1177 | 18.7811 | 0.7552 | 0.6696 | 0.0855 |
| **R32** | 8.7328 | 0.7673 | 0.7120 | 0.0552 | 2.0949 | 0.7673 | 0.7538 | 0.0135 |
| **R16** | 5.8626 | 0.7821 | 0.7291 | 0.0530 | 1.1149 | 0.7821 | 0.7719 | 0.0102 |
| **QF** | 4.1788 | 0.7779 | 0.7195 | 0.0584 | 1.2722 | 0.7779 | 0.7596 | 0.0183 |
| **SF** | 0.8482 | 0.7378 | 0.7201 | 0.0177 | -0.9497 | 0.7378 | 0.7582 | -0.0205 |
| **F** | -1.1932 | 0.6167 | 0.6627 | -0.0461 | -1.7407 | 0.6167 | 0.6869 | -0.0702 |

Figure 5: T-Tests for Slams-Masters Gap

For instance, R128 and R64 had slightly reduced gaps compared to the unadjusted. Meanwhile, R32 to the QF witnessed a much greater decrease in the gaps. Now, the adjusted Slams-Masters gap for R16 and QF are no longer statistically significant, as shown in Figure 5. Lastly, the reversal of the Slams and Masters lines occurred earlier, now in the semifinals of the adjusted graph. The interpretation for this earlier reversal could be that the stronger player is more likely to win best-of-5 set matches at the semifinals of a Masters tournament than a Slam because of lower pressure, which may cause improved relative performance. Figure 5 shows the sample t-tests of the difference in means between the Slams and the Masters for both the unadjusted and the adjusted Slams-Masters gap.

#### 4.2.3 Closing the Gap with Prize Money

Taking the difference between the Grand Slams and "Masters Adjusted" lines yields the newly adjusted Slam-Masters Gap. This adjustment pertains to great importance to the question posed in the beginning: "how much more prize money is needed in Masters tournaments to make stronger players try equally hard as in Grand Slams?". To this end, I multiply the prize spread of each match of a given round by a certain factor. It is impor-

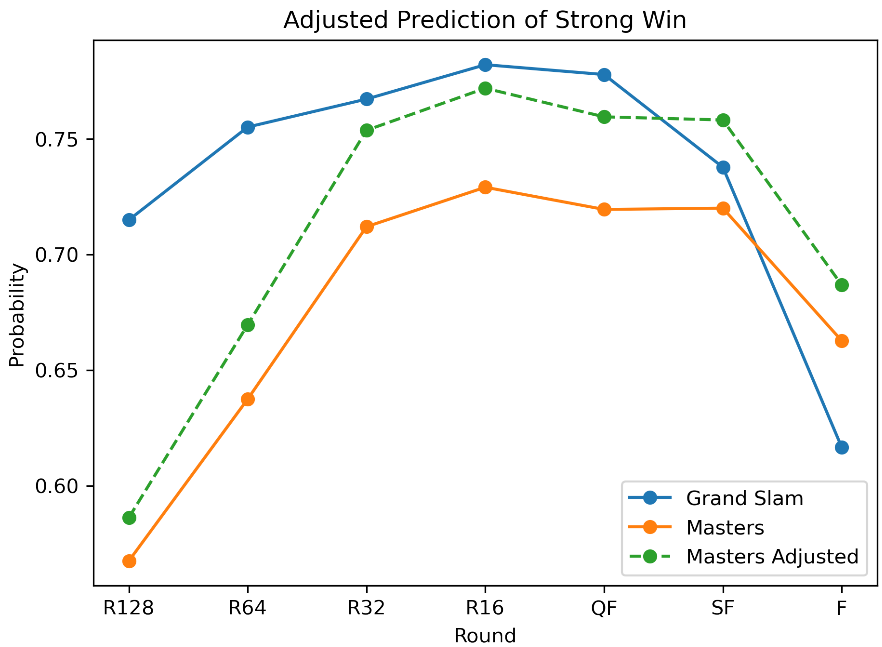



Figure 6: Adjusted Prediction of Stronger Player Winning Match

tant to note that our prediction of prize money on strong wins is monotonic, which means increasing the prize spread will make the stronger player more likely to win, given in our logistic regression odds ratios from Table 2. With the prize spread multiplied by a factor, I re-predict all observed matches of a given round and use an algorithm similar to goal seek to search for the optimal multiplicative factor to approximately "close the gap" in Figure 6. Table 4 presents the results from this exercise. Multiples greater than 1 suggest Grand Slams have a higher probability of the stronger player winning since this means that prize money in Masters tournaments needs to be increased to equalize the probabilities. It is important to note here that these multiples should not be interpreted causally as neither the unadjusted nor adjusted Slams-Masters gaps can be attributed to being driven by prize money alone.

Table 4: Masters Prize Money Multiple to Close the Gap

| Prize Multiple in All Observed Matches | | |
|---|---|---|
| Round | Unadjusted | Adjusted |
| R128 | 115.50 | 59.70 |
| R64 | 68.40 | 19.60 |
| R32 | 9.30 | 1.68 |
| R16 | 9.35 | 1.50 |
| QF | 11.80 | 2.09 |
| SF | 2.04 | 0.46 |
| F | 0.19 | 0.083 |

# 5 Discussions and Extensions

The results from the primary logistic regression and the robustness check presented in this paper are very promising and go a long way in describing the impact of monetary incentives on the probability of the stronger player winning and observed differences between tournaments of different prestige levels. The results show that the net effect of increasing the spread of the winner's prize and the loser's prize in any given round will benefit the stronger player on a magnitude between 15% (using current rank ratio) to 40% (using absolute heterogeneity) on average for each log point increase in the prize money. Another empirical phenomenon is the Slams-Masters gap, which records varying size differences in the probability of the stronger player winning by round. It is important to note that the analyses in this paper do not study effort per se but instead study the factors that Tournament Theory predicts to incentivize effort, which impacts match outcomes. It is more appropriate to think of monetary prize incentives as a potential factor that increases the motivation of both players to win in a given match. The incremental effort resulting from both players' greater motivation to win benefit the stronger player more. Even the robustness check, which only adjusts the Slams-Masters gap for the difference in play format, cannot isolate all the potential factors driving player behavior, which will impact the probability of the stronger player winning the match. In short, it is better to view the covariates as a broad set of historical skill-based factors and extrinsic incentives that impact match outcomes rather than factors that directly impact effort in and of itself, which is much more challenging to measure directly.

Many more lingering questions remain after the results of this paper. For one, there are still many behavioral factors that the current models do not encompass. One such example is the correlation or momentum between player performance between sets (which would violate our independence assumption in the robustness check), matches, and even between tournaments. Other variables cited in theory also impact how a tournament plays out and include the player's self-assessment of future competition and expected effort (similar to Jennifer Brown's shadow of future competition), a player's evolving self-belief in his prospects of continuation, and systematic biases (such as overconfidence) leading to skewed assessments of the relative abilities of his fellow competitors.

There have also been analyses of tennis psychology and performance from top players, particularly a phenomenon called "choking under pressure". Many studies have shown that the implicit cost of seeing the winner's trophy near the court during the finals will significantly decrease the performance of highly-ranked players. Further direction for improving the models inside this paper is to study the driver factors leading to the massive drop off of the probability of the stronger player winning that occurs in the finals and why it affects

Grand Slams more severely. Tournament Theory predicts that an increase in prize spread is conducive to tournament selection ability. This result seems to hold for the first half of tournaments from R128 to R16, but it is less clear what is causing the increased likelihood of upsets after that stage. The decrease in tournament heterogeneity in later rounds, tournament fatigue, and choking under pressure could all be potential factors. These are all open questions still to be explored.

Another general area of study in behavioral economics of great relevance in this paper is whether financial incentives, a form of an extrinsic incentive, will "crowd out" intrinsic incentives and decrease performance, which is the opposite prediction from Tournament Theory. Although I found no suspicions of this in this paper's analyses of tennis tournaments, it would be fascinating to examine this problem for tournaments with vastly different selection structures. Possible examples being the ATP World Tour Finals, a highly competitive tournament with the top 8 tennis players every year, which utilizes a Round-Robin approach in the group stage instead of single elimination. This analysis can also be applied into team-based sports such as soccer, with one application being the Chinese Super League. Many disappointed fans of the league blame undesevered contract salaries for mediocre players as the chief reason that demotivates player effort and prevents progress since money can be readily earned without expending much effort. Additionally, the influx of international superstars brought into Chinese teams does not significantly improve the league's competition. It would be exciting to examine whether the superstar effect also exists within teams, especially if the superstar is a teammate.

### 5.0.1 Bonus Model of Effort Provision

An idea for further study is the decision-making process of tennis players making dynamic decisions about the allocation of effort between and within tournaments. It is often mentioned in the media that athletes need to conserve their energy and deploy it effectively in crucial stages to extend the longevity of their career and perform well in the long run, especially in crucial moments. An economic framework for this idea of effort provision and conservation is to view the choices of tennis tournaments to participate in and how much effort to exert in these events throughout the tennis season as a constrained optimization problem. Players make dynamic decisions and choose to allocate and conserve varying amounts of physical effort from a finite budget within and across different tournaments, with the end goal to maximize prize money over the year.

One similar framework to reference is the Single-Agent Risk-Incentive Model from labor economics. This principle-agent standard agency model from Malmendier (2007) will be of great relevance when considering the above simulation exercise. In particular, Malmendier incorporated the idea of "luck", which has a random value drawn from a given distribution, as the variable of integration in the expected utility function of a worker's effort provision and compensation. As professional sporting results also have a strong luck component at certain times, the insights from Malmendier's paper would be very insightful going forward. Now, I will conclude the section by posing two questions as extensions to the analyses in this paper:

**1. Effort allocation in the run-up to a prestigious Grand Slam**. It is often seen as sub-optimal to exert maximum effort in a less critical tournament right ahead of a critical Grand Slam. Participating in tournaments comes with costly effort that could induce lasting fatigue, negatively impacting Grand Slam performance. The question becomes: how would players with different parameters of reactions to incentives and effort budgets make participation and effort provision decisions in this scenario?

**2. Effort provision and response to dynamic incentives within a tournament**. It is well documented in Tournament Theory that prizes at the top have a profound ripple effect that impacts decisions made in much earlier rounds. Even though the prize increase does not happen until multiple rounds later, players will likely try very hard even in the first round to maximize the chance of getting to the stage to compete for that large prize directly. Also, it could be intuitive that the closer the player is to the stage with the substantial payoff increase, the greater the effort that will be exerted, as the player's self-assessed probability of continuation increases. However, the question remains: how should effort be allocated across earlier rounds in anticipation of a significant prize spread increase that only happens in a much later round?

# 6 Conclusion

In conclusion, tournaments are a prevalent type of selection mechanism and reward structure to incentivize competition. Competitive contests have many widespread and powerful applications ranging from sporting events, promotions within a company, and political primaries. This paper studies the impact of monetary incentives in two-player single-elimination events of differing prestige levels and play formats. There are still many areas for research that can soundly incorporate behavioral phenomena into the decision-making of effort conservation to study the complexities of multi-stage dynamic and strategic contests. Research going forward in this direction will enhance our understanding of tournament-induced competition and can lead to remarkable improvements in the design of tournament structures.

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