Public Debt and Economic Growth: Is There a Change in Threshold Effect?

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Abstract

My paper investigates the relationship between public debt and GDP growth for European countries and the heterogeneous debt threshold effect before and after the financial crisis, as well as between countries with high and lower credit rating, by using a large cross sectional panel data and two estimation methods. My empirical results suggest that the short run impact of debt on GDP growth is positive and statistically significant but decrease to negative and lose some significance at public debt and GDP ratio of 109%. After the financial crisis, either the threshold effect vanishes or the threshold values increase based different cohort and method. Furthermore, countries with higher credit rating have higher threshold value and more significant threshold effect.

1 Introduction

Majority of the literature from the last decade discovers a negative correlation or an inverted U-shaped relationship between public debt and economic growth. When debt is low, increase in debt will encourage economic growth; however, when debt reaches some threshold value, further increase in debt would lower GDP growth. In the pioneer paper *The Impact of High and Growing Government Debt on economic Growth: An Empirical Investigation for the Euro Area*, Reinhart and Rogoff estimated a threshold value of 90% where above growth prospects are severely undermined. However, there remains debate regards to what the threshold value is.

In the aftermath of global financial maelstrom in 2008, soaring public debt in many European countries have outdistanced the threshold of 90%. According to the debt threshold effect, those countries should have significant decline in growth if the GDP growth rate if not negative. However, the economic growth of European countries with high credit rating are not much different as what they have before the financial crisis in 2008 and when their public debt to GDP are below or around the threshold of 90%. At the same time, GDP growth rate of European countries with low credit rating are indeed hampered (See Figure 3.3 for details). There are two natural questions to ask: 1) Whether financial crisis in 2008 made European countries more tolerance to debt, and 2) Are European countries with lower credit rating more responsive to debt.

The remainder of the paper is organized as follows. In Section 2, review relevant literature regarding public debt and economic growth rate with a focus on debt threshold. In Section 3, describe the data, the set-up and the methodology. In Section 4, give estimation results and test results. In Section 5, provide result from robustness checks, including check for influential countries and an analysis on impact of debt on long-term interest rates. Section 6, conclusion and further research. Appendix 1, data description and data summary. Appendix 2, regression results. Appendix 3, figures.

2 Literature Review

2.1 Theoretical Literature

Most of the theoretical literature conclude a negative relationship between public debt and economic growth. In *Dividend Policy, Growth, and the Valuation of Shares*, Merton H. Miller and Franco Modigliani (1961) conclude that the external debt leads to lower stock of private capital, which reduces flow of income and lowers growth. Peter Diamond (1965), in *National Debt in a Neoclassical Growth Model*, argues that when in the efficient case (interest rate is higher than growth rate), national debt negatively affects growth

by increasing tax burden and move interest rate way from the Golden Rule, and thus leads to a decrease in savings and consumption of taxpayers, which then lowers capital stock. However, when growth rate is higher than interest rate, indicating an over accumulation of capital, an increase in national debt improves allocation, and thus is beneficial to growth. In *Fiscal Policy in an Endogenous Growth Model*, Saint-Paul (1992) built on the growth model from Blanchard (1985) and Weil (1989) and analyzed the impact of fiscal policy on growth and found that, in an endogenous growth model, an increase in national debt reduces economic growth. David Aschauer (2000), in *Do States Optimize? Public Capital and Economic Growth*, proposed a growth model in which public debt has a non-linear (inverted U shape) relationship with economic growth and found that high level of national debt adversely affects growth rate. From the result of theoretic literature on national debt and growth, a logical question to ask is whether growth always negatively affected by debt or is there a debt threshold above which debt becomes a burden on growth. John Cochrane, in *Inlfation and debt* The public debt can negatively impact growth because increases uncertainty or leads to expectations of future confiscation, possibly through inflation and financial repression (see Cochrane, 2011a,b, for a discussion of these issues). In this case, higher debt could have a negative effect even in the short-run.

2.2 Empirical Literature

However, this question is still in dispute in existing empirical literature. I will first review three influential papers in detail. Then, I will discuss the disagreements from major literature regarding this topic.

The most influential paper regarding debt and growth is *Growth in a Time of Debt* published in 2010 written by Carmen Reinhart and Kenneth Rogoff. The paper used an empirically approach, using a panel of 3700 annual observations of 44 countries spanning about two hundred years from 1790 to 2009. They classified the regimes into four categories: advanced economies with low national debt to GDP ratio (below 30%), medium-low national debt to GDP ratio (between 30% and 60%), medium-high national debt to GDP ratio (between 60% and 90%), and high national debt to GDP ratio (above 90%), and conclude that countries with high public debt have significantly lower growth rate. Although the large data enables readers a have panoramic view, the lack of incorporating different country's specific factors such as political system and trade openness leads to the issue of heterogeneity. Also without using econometric and instrumental variables, the results did not prove any causal relationship between debt and growth. Nevertheless, their findings have inspired many scholars to research regrading this topic and evaluate their results. Later researches on this topic have used econometric and multivariate regression to control endogeneity, reverse causality, and cross-country heterogeneity.

The Impact of High and Growing Government Debt on economic Growth: An Empirical Investigation for the Euro Area written by Cristina Checherita and Philipp Rother in another influential paper on this topic that focused on only developed European countries. They used data mainly from the European Commission AMECO database of 12 developed European countries from 1970 to 2011. Thus, the issue of heterogeneity, which is often problematic in growth regressions, is alleviated by using a relatively restricted cross-sectional sample. They used an empirical growth model which is built on a conditional convergence equation that relates the GDP per capita growth rate to the initial level of GDP per capita, the investment/saving-to GDP rate, and gross government debt as a share of GDP. To control the issue of endogeneity, they instrument the debt variable for each country through either its time lags (up to lag of order 5) or through the mean debt levels of the other countries in the data. They conclude that the debt-to-GDP threshold of this inverted U-shape relationship is roughly between 90% and 100% on average for the data.

A similar paper *Debt and Growth: New Evidence for the Euro Area* written by Anja Baum, Cristina Checherita-Westphal, Philipp Rother (BCR) used a slightly different approach. They also used annual data of 12 developed European countries but from period 1990 to 2010. They first used the dynamic panel threshold model and 2SLS to find the threshold value that minimizes the residuals. Then they used the benchmark model which regresses GDP growth on short term interest rate, debt to GDP ratio, openness to trade, ratio of gross capital formation and EMU membership to test the relationship between debt and growth and the significance their result. The issue of heterogeneity and endogeneity is control using the method stated in the previous paper. They found that for high debt ratios (above 95%) the impact of additional debt has a negative impact on growth rate.

In *The real effects of debt*, Stephen Cecchetti, M Mohanty and Fabrizio Zampolli (2011), report a debt threshold of 85% of GDP by using data from 1980 to 2010 of 18 OECD countries and perform a growth regression on control variables such as debt to GDP ratio, gross saving, inflation, population growth and openness to trade. Pier Padoan, Urban Sila, and Paul Noord (2012), in *Avoiding Debt Trap: Financial Backstops and Structural Reforms*, using a panel of 28 OECD countries over 1960 to 2011 and similar growth equation, discover a similar debt threshold of 90%. However, Balazs Egert (2015), in *Public Debt, Economics Growth and Nonlinear Effects: Myth or Reality?*, using the dataset and the four-regime model of RR (2010) and a bivariate regression, discovers a much lower debt threshold of 20% and 60% above which GDP growth prospects are severely undermined. Nevertheless, Chang and Chiang (2009), in *Transitional Behavior of Government Debt Ratio on Growth: Case of OECD Countries*, using data of 19 OECD countries over the period 1993-2007 and a panel smooth transition regression model that controls heterogeneity across countries, find a significantly positive relationship between one-year lagged government debt ratio and real GDP across all regimes. However, Alfredo Schclarek (2004), *Debt and Economic Growth in Developing and*

Industrial Countries, reports only a negative relationship between debt and growth for developing countries but not for developed countries using a panel of 54 emerging markets and 29 advanced economies between 1970 and 2002 and performing linear and nonlinear regression on growth. Similar result was found by Andrea Pescatori, Damiano Sandri, and John Simon, in *Debt and Growth: Is There a Magic Threshold?*. They used 36 developed country, covering period from 1821 to 2011. Using method similar to RR's, they conclude that there is no debt ratio threshold above which an adverse relationship between debt and growth kicks in.

I contribute to the current strand of literature on public debt and economic growth by examine possible change in threshold effect from financial crisis and difference in response to debt for European countries cohort by credit rating. My result is robust as I controlled for omitted variable bias and reverse causality. This paper makes a unique contribution to the debate on debt threshold by presenting new empirical evidence on the heterogeneity across European countries cohort by year and credit rating based on a sizeable dataset.

3 Data and Methodology

3.1 Data

Data used in this paper was extracted from the European Commission Database (AMECO). The panel data consists observations over year 1988-2018 including 30 European Countries (See Appendix 1 for details). By using a relative short time span, my data is less prone to political and economic structural changes and more comparable with today's economic settings. Given the data encompasses large number of countries, country fixed effect is used to control for cross-section heterogeneity. Because of the relatively unconstrained cross-section data, my result can be susceptible to outlier countries with extremely low or high debt level. Regression result with outlier countries removed is included in robustness check. In this paper, GDP growth rate is the main dependent variable. Indicator of whether the country's debt to GDP ratio excess some threshold value and debt to GDP ratio are the two main variables of interest.

3.2 Regression and Variable Construction

One should note that in section 3, same letter may be used to represent a coefficient in different equation. This does not mean estimated coefficients are the same. The coefficients in each equation are estimated uniquely from regression.

3.2.1 Initial Estimation Equation and Possible Issue

Using variables of interest, the two initial estimation equations are in the form of

$$GROWTH_{i,t} = \alpha_i + \beta_1 I(d_t > d^*) + e_{i,t} \tag{1}$$

where $GROWTH_{i,t}$ is the one year GDP growth rate of country i in year t, α_i is a constant, $I(d_t > d^*)$ is an indicator which equal 1 if the country i has debt to GDP ratio ($d_{i,t}$) is greater than the threshold d^* and 0 otherwise, and $e_{i,t}$ is residual for country i at year t.

$$GROWTH_{i,t} = \alpha_i + \beta_1 d_t + +\beta_2 {d_t}^2 + e_{i,t} \tag{2}$$

where $d_{i,t}$ is the current debt to GDP ratio $d_{i,t} - d_{i,t-1}$, ${d_{i,t}}^2$ is added to capture the possible non-linear effect of debt on economic growth, other variables are the same as as from the first initial estimation equation (equation (1)).

The above two equations clearly suffer from reverse causality and omitted variable bias. Reverse causality happens when the dependent variable also effects independent variable, making the result not casual. To deal with reverse causality, instrumental variable need to be used. Omitted variable happens when that regression excludes a variable that is correlated with other covariates and the dependent variable. Consider an simple OLS regression,

$$Y_1 = \alpha + \beta_1 X_1 + u \tag{3}$$

whereas the correct model should be,

$$Y_1 = \alpha + \beta_1 X_1 + \beta_2 X_2 + v \tag{4}$$

Then, $u = \beta_2 X_2 + v$. Therefore,

$$\hat{\beta_1} = \beta_1 + \beta_2 \times \frac{cov(X_1, X_2)}{var(X_1)} + \frac{cov(X_1, v)}{var(X_1)} \tag{5}$$

Then if $\beta_2 \neq 0$ and $cov(X_1, X_2) \neq 0$, $\hat{\beta_1}$ will be biased. The direction of bias depends on the sign of β_2 and $cov(X_1, X_2)$. To deal with omitted variable bias, control variables need to be added.

3.2.2 Variable Selection and Construction

To mitigate the issue of reverse causality, a internal instrument of one year lagged debt to GDP ratio ($d_{i,t-1}$) is used. One should be aware, however, this instrument does not eliminate reverse casualty. The advantage of using this internal instrument compare to an external instrument is that the internal instrument provides a more direct and interpretable result. To measure the possible non linear effect, squared one year lagged debt to GDP ratio (${d_{i,t-1}}^2$) is also included in the final estimation.

To avoid omitted variable bias, country fixed effect, population growth and size of government, approximated by total government expenditure to GDP ratio, and lagged GDP growth are construed and included in the final estimation. Ideally, year fixed effect should be included to capture the possible difference in economic condition across year. Interaction terms between dummies for year and one year lagged debt to GDP ratio and between dummies for country and one year lagged debt to GDP ratio should also be added to capture the heterogeneous effect of debt on economic growth across year and across country. However, because my data includes 30 countries and spans 30 years, including above variables will introduce too many coefficients which need to estimated relative to the sample size. This will introduce large variance and lower estimation precision. Therefore, I shall not include year fixed effect, interaction terms between dummies for year and one year lagged debt to GDP ratio and between dummies for country and one year lagged debt to GDP ratio. By doing this, I am posing the assumptions: 1)Economic condition are similar across year (exclude year fixed effect), 2)Responses to a increase in debt is the same across year (exclude interaction between year dummy and debt), and 3) Countries have the same response to a increase in debt (exclude interaction between country dummy and debt). However, it is possible that economic condition and response to a increase in debt changes after financial crisis. Therefore, to capture the impact of financial crisis on the effect of debt on growth, I included fixed effect of financial crisis by adding a dummy variable which equals to 1 if year is greater than or equal to 2008 and 0 otherwise and interaction between this dummy and debt.

I shall not elaborate on the significance of country fixed effect, population growth and size of government here (Support of this can be found from other literature). I will, however, explain the reason and possible issue with including lagged GDP growth. If current GDP growth and current debt to GDP ratio are negatively correlated. Then excluding lagged GDP growth will lead to a downward bias on the coefficient for debt To find the optimal year of lags, Autoregressive model of the form below is used.

$$X_{i,t} = \mu_i + \beta t + \phi_{i,t-1}X_{i,t-1} + \phi_{i,t-2}X_{i,t-2} + ... + \phi_{i,t-p}X_{i,t-p} + Z_{i,t} \tag{6}$$

where $X_{i,t}$ is GDP growth rate for country i at year t, μ_i and t are deterministic term (constant and trend), and $Z_{i,t}$ is residual for country 1 in year t assumed to have zero mean and constant variance σ^2. I choose

$p = 1, 2, 3, 4, 5$ because of the constrained time span of my data. The optimal level of lag p is chosen by: 1) Model selection method such as Akaike information criterion(AIC) and Baysian Information criterion (BIC). 2) t-test on each lagged variable and F-test on all lagged variable. For method 1): for a selection of model M_1, M_2, ..., M_k, where M_k has k parameters, select the model which minimizes

$$AIC_K = -2log(\hat{L_k}) + 2K, \text{ where } \hat{L_k} \text{ is the maximum likelihood of } M_k \tag{7}$$

and

$$BIC_K = -2log(\hat{L_k}) + K \times log(n), \text{where } \hat{L_k} \text{ is the maximum likelihood of } M_k \text{ and n is the sample size} \tag{8}$$

Both AIC and BIC choose a model with higher maximum likelihood while controlling for the number of parameters (issue of overfitting). However, one should note, that BIC tends to select a model with fewer parameters because of the larger punishing term $k \times log(n)$ compare to AIC's punishing term 2k. For method 2): Both t test and F test need the assumption that residuals are distributed as $N(0, \sigma^2)$. T-test tests the hypothesis that $\phi_i = 0$ against its two sided alternative that $\phi_i \neq 0$. F test tests the null hypothesis that $\phi_i = \phi_J = ... = \phi_m = 0$ against its two sided alternative that the null hypothesis is not true. If p-values is large ($> 10\%$) resulting from t-test, then covariates corresponding to ϕ_i does not have significant predicting power on the dependent variable, and thus can be exclude from the regression. Similarly, if p-values is large ($> 10\%$) resulting from t-test, then one or more of covariates corresponding to $\phi_i, \phi_j, ..., \phi_m$ jointly does not have significant predicting power on the dependent variable, and thus some can be exclude from the regression. For majority of countries, using just one year lagged GDP growth rate gives the lowest value of AIC and BIC. Furthermore, for majority of countries, the only significant variable is one year lagged GDP growth rate; p-value from F test increase significantly after adding p^{th} order lag for $k > 1$. This means that longer lags of GDP growth have relatively small predicting power on current change in GDO growth compare to on year lagged GDP growth. Result using debt to GDP ratio as dependent variable (regress on lagged growth rate of GDP) is analogues. I only include one year lagged GDP growth rate in the final regression. One possible issue with including one year lagged GDP growth rate is if correlation between this variable and change in debt to GDP ratio is extremely high, then the precision of the estimated regression coefficients decreases. However, highly correlated covariates does not appear to be an issue.

3.2.3 Final Estimation Equations

The first estimation equation for the threshold value is in the form,

$$GROWTH_t = \alpha + \beta_1 I(d_{t-1} > d^*) + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 GROWTH_{t-1} + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + \phi_4 I(POST2008) + \phi_5 I(POST2008) * I(d_{t-1} > d^*) + U_t \quad (9)$$

where $GROWTH_{i,t}$ is the one year GDP growth rate in year t, α constant, $I(d_{t-1} > d^*)$ is vector of an indicators which equal to 1 if one year lagged debt to GDP ratio is above the debt threshold and 0 otherwise, $I(country_i)$ is vector of an indicators which equal to 1 if country is $country_i$ and equal to 0 otherwise (n is the total number of country in the data), d_{t-1} is a vector of one year lagged debt to GDP ratio, $POPGROWTH_t$ is a vector of population growth at time t, $GOVSIZE_t$ is a vector of size of government at time t, approximated by total government expenditure to GDP ratio, $I(POST2008)$ is an vector of indicators which equals to 1 if year is greater than or equal to 2008 and 0 otherwise, $I(POST2008) * I(d_{t-1} > d^*)$ is a vector of interaction terms between a vector of indicators for post financial crisis and a vector of indicator for debt to GDP ratio above the threshold, which intends to capture the different effect of debt threshold on economic growth before and after the 2008 financial crisis, and U_t is a vector of residuals assumed to have zero mean and jointly distributed as $N(0, \Sigma)$. Threshold value(d^*) is chosen by two procedures: 1) "Coefficient Method": Choose the threshold value such that itself has a negative coefficient, any smaller threshold value has positive coefficient, and any large threshold value has a negative coefficients. 2) "Minimizing RSS Method" : Find the threshold value that minimizes the residual sum squared.

The second estimation equation for the threshold value is in the form,

$$GROWTH_t = \alpha + \beta_1 d_{t-1} + \beta_2 {d_{t-1}}^2 + + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 GROWTH_{t-1} + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + \phi_4 I(POST2008) + \phi_5 I(POST2008) * d_{t-1} + U_t \quad (10)$$

where d_{t-1} is one year lagged debt to GDP ratio, $I(POST2008) * d_{t-1}$ is a vector of interaction terms between a vector of indicators for post financial crisis and a vector one year lagged debt to GDP ratio above the threshold, which intends to capture the different effect of debt on economic growth before and after the 2008 financial crisis, and other variables are the same as from the the first estimation equation (equation (8)). The threshold value is chosen by calculating the turning point from coefficients for one year lagged debt to GDP ratio (d_{t-1}) and squared one year lagged debt to GDP ratio (${d_{t-1}}^2$) in the following method

(solution to a polynomial of degree 2),

$$turning\ point = -\frac{\beta_1}{2 \times \beta_2} \tag{11}$$

where β_1 is the coefficient for d_{t-1} and $beta_2$ is the coefficient for ${d_{t-1}}^2$.

4 Regression Result and Discussion

One should note that in section 4, same letter may be used to represent a coefficient in different equation. This does not mean estimated coefficients are the same. The coefficients in each equation are estimated uniquely from regression.

4.1 Regression Result

For the first estimation equation, I run regression for every integer value between 0 and 130 as the threshold value (d^*) and obtained coefficient corresponds to each $I(d_{t-1} > d^*)$ (Table 2.1). Using full set of data, I do observe the threshold effect, where the coefficients have a decreasing trend as threshold values increase and eventually become negative (Figure 3.4). The coefficient decrease dramatically from positive to negative at debt to GDP ratio of 109% and further decrease as threshold value increase. However, the coefficient become less negative when threshold value increase to 127%. The coefficient of $I(d > d^*)$ corresponds to $d^* = 109\%$ is -0.876. The result is significant at 5% level. This suggests a robust result that when everything else is the same, country with debt to GDP ratio above the threshold value 109% experience 0.876% lower growth rate than country with debt to GDP ratio below the threshold. However, applying "the minimizing RSS" method on the first equation gives my lower estimated threshold values (10 %-20%). This can be explained by the dramatic decrease in coefficient significant when threshold value is large, which means that when debt is sufficient high, government debt has significantly lower predicting power on economic growth. Therefore, minimizing RSS tends to give lower debt threshold value where the predicting power on debt is still high. Estimated threshold value from minimizing RSS does not mean country with debt to GDP ratio above this threshold will experience negative economic growth. Therefore, the "minimizing RSS method" can viewed just as a optimization method with less economic interpretation compare to the "coefficient method". For this reason, I shall only consider "the coefficient method" in the following paper.

Based on the second estimation equation, one year lagged debt to GDP ratio and squared one year lagged debt to GDP ratio are also variable of interest (Table 2.2). The estimated coefficient for the two variables

are 0.126 and 5.46×10^{-4} respectively. Both coefficients are significant at 0.1% level. This suggest that a 1% increase in debt to GDP ratio will result in a 0.126% increase in economic growth rate in the following year. However, because of the coefficient for squared one year lagged debt to GDP ratio is negative, the positive effect of debt to GDP ratio on economic growth rate decreases as debt increase. The turning point can be calculated using equation 11 which gives a estimate for the threshold value of 115%.

Both regressions give similar estimations for the threshold value which are higher than 90% found by Reinhart and Rogoff. Both estimation of threshold value suggest that countries become more tolerance to debt after the financial crisis. Using 90% as threshold value and run regression using equation 9, the estimated coefficient is 0.512. This suggest that country with debt to GDP ratio higher than 90% experience 0.512% more growth than country with debt to GDP ratio higher than 90%, which is contradicts to Reinhart's and Rogoff's result. Furthermore, coefficient for $(POST2008)$ and $I(POST2008) * d_{t-1}$ are significant at 1% level. Financial crisis in 2008 do seem to have impact on the effect of debt on economic growth rate. Therefore, I will estimate threshold value before and after the financial crisis to make this impact clearer.

4.2 Heterogeneity by Cohort

4.2.1 Year Cohort

The two estimation equations for investigating the heterogeneity by year cohort are,

$$GROWTH_t = \alpha + \beta_1 I(d_{t-1} > d^*) + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 GROWTH_{t-1} + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + U_t \tag{12}$$

and

$$GROWTH_t = \alpha + \beta_1 d_{t-1} + \beta_2 {d_{t-1}}^2 + + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 GROWTH_{t-1} + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + U_t \tag{13}$$

where variables in equation 12 is the same as in equation 9 and variables in equation 13 is the same as in 10 but with $I(POST2008)$ and $I(POST2008) * I(d_{t-1} > d^*)$ removed in equation 12, and $I(POST2008)$ and $I(POST2008) * d_{t-1}$ removed in equation 13.

Before the financial crisis, the estimated threshold value from equation 12 and 13 are 71% and 76% respectively (Figure 3.5, Table 2.1, Table2.2). The coefficient for $I(d_{t-1})$ is -1.27 which is higher than the estimated coefficient of -0.876 using the full data set. The estimate is also significant at % level. This lower and significant threshold value combined with the large magnitude estimated coefficient suggest that

countries are more sensitive to debt before the financial crisis.

After the financial crisis, equation 12 does not give an estimation for threshold value because coefficients on $I(d_{t-1})$ are all positive for threshold value between 30 and 150 (Figure 3.6, Table 2.1, Table2.2). However, I do observe a decreasing trend in economic growth rate as debt threshold increase. Using the threshold value 109% found by equation 9, the estimated coefficient for $I(d_{t-1})$ is 0.375. This suggest that keeping everything else the same, after the financial crisis, country with debt to GDP ratio above 109% experience 0.375% higher growth rate than country with with debt to GDP ratio less than or equal to 109%. Equation 13 also does not give a practical estimation of threshold value. Furthermore, coefficients for variables of interest from equation 12 and 13 are not significant at 10% level. This suggest not only country become more tolerance to debt, but also debt has less predicting power on economic growth rate after the financial crisis.

4.2.2 Credit Rating Cohort

Credit rating can effect threshold value through its effect on interest rate. Higher credit rating is associated with lower risk, and therefore, lower risk premium. Then high credit rating country should theoretically experience less increase in interest rate, and thus less impact on economic growth. To investigate the effect of credit rating on threshold value, I ran regression using equation 9 and 10 on country with low credit rating and high credit rating separately. Using S&P crediting rating, I classify a country as high credit rating if it receives rating better or equal to AA, and country has low credit rating if it has rating worse than all equal to BB. According to this classification method, low credit rating countries are Bulgaria, Croatia, Cyprus, Hungary, Italy, and Romania. High credit rating countries are Austria, Germany, Sweden, Netherlands, Norway, Luxembourg, Belgium, Denmark, Finland, and France. One should note that this classification method will significantly reduce sample size. Also, because country with low credit rating tends to be less developed, there are less data available. Therefore, the precision of estimates will be low for country with high credit rating and even worse for low credit rating country.

For low credit rating country, the estimated threshold value from equation 9 is 60% (Figure 3.7). However, estimated coefficient for $I(d_{t-1} > d^*)$ is insignificant with a value of -0.0415 (See table 2.1 for detailed result). For high credit rating country, the estimated threshold value from equation 9 is 115% (Figure 3.8). The estimated coefficient for $I(d_{t-1} > d^*)$ is -0.765 and significant at 10% level (Table 2.1). However, for low credit rating country, equation 10 does not give a practical estimated threshold value (Table 2.2). However, the result is not significant which may be due to limited sample size. For high credit rating country, equation 10 estimated a turning point at 119% debt to GDP ratio (Table 2.2). Because data is already limited using years from 1990 to 2018, I shall not further investigate the effect of financial crisis on debt threshold for low

and high credit rating country.

Although the estimated threshold value may not be precise, the relative magnitude of threshold values for low and high credit rating country can still shed light on the heterogeneity in debt tolerance. High credit rating country have significantly higher threshold value than low credit rating country. In addition, estimated threshold value for high credit rating country is also higher than the estimated threshold value when using full data set. Therefore, one may conclude that high credit rating country are more tolerance to debt. However, further analysis should be done to validate this result.

5 Robustness Check

One should note that in section 5, same letter may be used to represent a coefficient in different equation. This does not mean estimated coefficients are the same. The coefficients in each equation are estimated uniquely from regression.

5.1 Effect on Long Term Interest Rate

5.1.1 Result Using Full Data Set

Higher government debt is likely to be associated with higher long term interest rates because investors often connects high debt level with high sovereign risk premium. Therefore, large increase in debt can theoretically lead to higher long term interest rates which cause a decrease in private spending growth, and thus reduce economic growth. To investigate the effect of high government debt on long interest rate, the following two estimation equation are used,

$$\begin{aligned} {INT_t}^l = \alpha + \beta_1 I(d_{t-1} > d^*) + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 GROWTH_{t-1} \\ + \phi_2 {INT_{t-1}}^s + \phi_3 I(POST2008) + \phi_4 GOVSIZE_t + U_t \end{aligned} \tag{14}$$

where ${INT_t}^l$ is long term interest rate at year t, ${INT_{t-1}}^s$ is long term interest rate at year t-1, and other variables are the same as in equation 9.

Running regression using equation 14 with threshold value from 0% to 130%, the coefficient on $I(d_{t-1}$ have a increasing trend from negative to positive as the threshold value increases (Figure 3.9, Table 2.3). This means that if two countries both have low public debt then the country with higher public debt have lower interest rate. The coefficient become positive at threshold value of 103% and experience a dramatically increase at threshold value of 132%. This suggest that country with debt to GDP ratio above 103% will have

higher long term interest rate than country with debt to GDP ratio below or equal to 103%. One should note that this threshold value is slight below the estimated threshold from equation 9. This does not discredit the estimations because a increase interest rate does not necessarily lead to negative economic growth. The increase interest rate has to be sufficiently large to bring economic growth negative. Consider the following case, if a country's debt to GDP ratio just went above the threshold of 103% estimated by equation 14, a further increase in debt to GDP ratio is needed so that the increase in long term interest rate is sufficient large to bring economic growth negative. Then the threshold value which correlates with negative growth is higher than 103% estimated by equation 14. Then the discrepancy between threshold values estimated from equation 9 and 14 is even smaller. The estimated coefficient on $I(d_{t-1} > d^*)$ is 0.01 and insignificant when threshold value of 103% is used. This small coefficient validates my argument above because a 0.01% higher interest rate is too small to bring the economic growth negative. Using threshold value of 109% found from equation 9, the estimated coefficient on $I(d_{t-1} > d^*)$ is 1.56 and is significant at 0.1% level. This suggest that country with debt to GDP ratio above 109% experience 1.56% higher long term interest rate than country with debt to GDP ratio below 109%. This 1.56% higher long term interest rate is more likely to have a negative impact on economic growth than 0.01%. However, further analysis needs to be done to estimate a long term interest rate which brings economic growth negative.

5.1.2 Effect of Financial Crisis

Running equation on data before 2008, coefficients of $I(d_{t-1} > d^*)$ have a increasing trend from negative to positive as threshold value increase for threshold value below 40% (Figure 3.9). However, the coefficients for threshold value over 40% fluctuates around 0.196 (sometimes decrease to negative) (Table 2.3). The threshold value is hard to identify in this case. One could argue that the threshold value for which the coefficient change from positive to negative for the first time is 33%. This threshold value is significant lower than the threshold value estimated using equation 9 on data before 2008. The coefficient on $I(d_{t-1} > d^*)$ associated with this threshold value is 0.0273%. This suggests that before the financial crisis, country with debt to GDP ratio below or equal to 33% experience a 0.0273% higher long term interest rate. This is relatively small compare to the coefficient of 1.56 estimated in previous subsection using threshold value of 109%. The small coefficient corresponds to threshold value of 33% and the pattern of coefficient for threshold value above 40% may explain the extremely low threshold value here. Because difference in long term interest rate for country with debt to GDP ratio above and below threshold value from 33% to 130% is only 0.197 on average, the country need to significantly increase government debt so that the increase in long term interest rate is sufficient enough to bring economic growth negative. If this is the case, the debt to GDP ratio for this country would be a lot higher than 33%. The small fluctuating coefficients mean that, for country with debt

to GDP ratio above threshold 33%, an increase in debt to GDP ratio has small and ambiguous effect on long term interest rate. This may suggest that an increase in debt to GDP ratio has small effect on risk premium. Therefore, before the financial crisis, investors may be confident that even country with high debt has the ability to pay its debt. One may think that my result for threshold before financial crisis contradicts my finding in section 4.2.1 that threshold value is lower before financial crisis. Based on my finding, one would expect debt threshold to be higher before the financial crisis because increase in debt does not translate to large clear increase in long term interest rate, then large increase in debt is needed for a large increase in long term interest rate so that a negative growth rate is possible. However, this discrepancy merely suggests that there are other channels for which increase in debt effects economic growth.

Running equation on data after 2008, coefficients of $I(d_{t-1} > d^*)$ have a increasing trend from negative to positive as threshold value increase (Figure 3.11). The estimated threshold is 130% for which any threshold value above have positive coefficients. This is largely in line with my result from section 4.2.2 that no threshold value is found using data after 2008. The coefficient on $I(d_{t-1} > d^*)$ using threshold value of 130% is 0.229 (Table 2.3). Although this coefficient is not large, the coefficient increase dramatically to 4.65 for threshold value of 132%. This explains the large threshold value because for country with debt to GDP ratio above the threshold of 130%, any further increase in debt has a dramatic increase in interest rate that is likely to bring economic growth rate to negative.

Overall, my result from this section largely in line with my finding in section in 4.2.1. Estimated threshold using equation 14 for data before the financial crisis is significantly lower than using data after the financial crisis. This increase in threshold value suggests that countries became more tolerance to debt after the financial crisis.

5.2 Effect of Outliers

To avoid my result being driven by country with too high or low debt to GDP ratio, I removed 1) Country outlier: country with both debt to GDP ratio and economic growth significant different from other countries and 2) Country with high leverage (statistically): country with extremely high or low debt to GDP ratio. Then I ran regressions using equation 9 and 10 excluding some countries identified by above method (Table 2.4, Table 2.5 respectively).

First, I excluded Estonia (country outlier) with on average extremely low debt to GDP ratio (5.18%) and low economic growth (4.20%). Using equation 9, the estimated threshold is 110% for which any higher debt to GDP ratio has a negative impact on economic growth. This threshold is slightly higher than 109% estimated using question 9 on the full data set. However, the coefficient corresponds to $I(d_{t-1} > 111\%)$ is

-0.310 which is smaller than -0.876 estimated using question 9 on the full data set. This means that difference in economic growth is very small for country with debt to GDP ratio below and above the debt threshold value. This result seems reasonable because Estonia also has the highest economic growth rate on average. Therefore, removing Estonia narrows the difference in economic growths between country with debt to GDP ratio below and above the threshold. The threshold estimated using equation 10 is 117% which is higher then the turning point of 115% estimated using equation 10 on the full data set. Overall, removing Estonia does not change my result significantly.

Second, I excluded Greece (country outlier) with significantly high debt to GDP ratio (130.09%) and low economic growth (0.7444%). The estimated threshold using equation 9 is 109% which the same as the threshold value estimated using equation 9 on the full data set. This result seems somewhat surprising because one would expect threshold value to decrease after removing Greece from the regression. However, after taking a close look at Greece's data, the estimations seems plausible. Greece experienced some slow positive and some negative economic growth when it has a debt to GDP ratio above 109%. Because the threshold is estimated by looking at the average growth for country below and above the threshold while controlling for other factors, it is probable that the effect of removing Greece vanishes after averaging. However if one construct more granular partition of threshold values, the effect of removing Greece may be more obvious. The coefficient of $I(d_{t-1} > 109\%)$ is -0.322 which is smaller than -0.876 estimated in section 4. This can be explained by the negative growth associated with debt to GDP ratio above 109% removed from regression. The threshold estimated using equation 10 is 107% which is lower then the turning point of 115% estimated using equation 10 on the full data set. This decrease in threshold value is reasonable because equation 10 captures the linear and non-linear effect 1% increase in debt to GDP ratio. Therefore, equation 10 captures more information from data than equation 9 which looks at the average. One should note that the two estimated thresholds are still higher than 90% which was estiamted by Reinhart and Rogoff. Overall, removing Greece does not change my result significantly.

Third, I excluded Estonia (country outlier) with on average extremely low debt to GDP ratio (5.18%) and low economic growth (4.20%), Luxembourg with on average extremely low debt to GDP ratio (14.99%), and Bulgaria on average extremely low debt to GDP ratio (21.41%). The estimated threshold using equation 9 is 128% for which any higher threshold value correspond to a negative coefficient. However, one should know that, the coefficient is also negative at threshold 119%, however coefficients correspond to threshold values between 119% and 128% are positive. The coefficient associated with threshold value 128% is -0.276 which is also lower than -0.876 estimated by equation 9 using the full data set. This means that difference in economic growth is relatively small for country with debt to GDP ratio below and above the debt threshold value. The estimated threshold using equation 10 is 117% which is higher then the turning point of 115%

estimated using equation 10 on the full data set. Overall, removing Estonia does not change my result significantly.

Fourth, I excluded Greece (outlier) with significantly high debt to GDP ratio (130.09%) and low economic growth (0.7444%), Italy (outlier) with significantly high debt to GDP ratio (113.14%) and low economic growth (0.564%) and Belgium (high leverage) with significantly high debt to GDP ratio (103.57%). The estimated threshold using equation 9 is 105% which is low than the threshold 109% estimated using equation 9 on the full data set. the coefficient correspond to $I(d_{t-1} > 105\%)$ is -0.374 which smaller than -0.876 estimated in section 4. This result is logical because Greece and Italy have the top 2 debt to GDP ratio and the bottom 2 GDP growth rate. Therefore, the economic growth rate for remaining countries with debt to GDP ratio above threshold 105% is not significantly lower than countries with debt to GDP ratio below threshold 105%. One should note that this threshold value is still higher than 90% estimated by Reinhart and Rogoff. The coefficient correspond to $I(d_{t-1} > 90\%)$ is 0.377, which means that on average, with other factors the same, country with debt to GDP ratio above 90% experience higher growth than country with debt to GDP ratio below 90%. The estimated threshold from equation 10 is 111% which is lower than 115% estimated in section 4. However, this estimation is not significant. This may be due to lower variance (spread) in debt to GDP ratio and GDP growth rate which makes the relationship less obvious. In conclusion, removing Greece, Italy and Belgium do result in a lower threshold estimation, but it is still higher than Reinhart and Rogoff's estimation.

Finally, I excluded all countries mentioned above. The estimated threshold value using equation 9 is 106% which is lower than 109% estimated by using the full data. The estimated coefficient for $I(d_{t-1} > 106\%)$ is -0.106 which is smaller than -0.876 estimated in section 4. This decrease in magnitude of coefficients is logical because countries with too high or low GDP growth and debt to GDP ratio were excluded. Using equation 10, the estimated threshold is 108%. However, both estimates are not significant which may be due to smaller sample size and lower variance (spread) in debt to GDP ratio and GDP growth rate.

For the most part, running regression excluding influential counties still give consistent results. Therefore, having outliers and data points with high leverage in my regressions do not incur substantial consequence; however the heuristic meaning of outliers generally shall be underestimated.

5.3 Growth of GDP per Capita as Dependent Variable

GDP per capita can also be used to investigate the relationship between debt and growth economic growth. To see this,

$$\frac{\Delta x/x}{\Delta t} \approx \frac{\partial x/x}{\partial t} = \frac{1}{x}\frac{\partial x}{\partial t} = \frac{\partial lnx}{\partial t} \approx \frac{\Delta lnx}{\Delta t} \tag{15}$$

Then for $\Delta t = 1$ and g^x be the growth rate x,

$$g^x = \Delta x/x = \Delta lnx \tag{16}$$

Also,

$$\frac{\Delta\ x/y}{x/y} = \Delta ln\frac{x}{y} = \Delta lnx - \Delta lny = g^x - g^y \tag{17}$$

Let x = GDP, y=population, then x/y = GDP per capita, then,

$$g^{\text{GDP per capita}} = g^{GDP} - g^{population} \tag{18}$$

Recall that variable $POPGROWTH$ in equation 9 and 10 is population growth rate, then estimates from equation below should give similar estimates as from equation 9.

$$\begin{aligned} GDPCAPITAGROWTH_t = \alpha + \beta_1 I(d_{t-1} > d^*) + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 Growth_{t-1} \\ + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + U_t \end{aligned} \tag{19}$$

Similarly, estimates from equation below should give similar estimates as from equation 10.

$$\begin{aligned} GDPCAPITAGROWTH_t = \alpha + \beta_1 d_{t-1} + \beta_2 {d_{t-1}}^2 + + \sum_{i=1}^{n-1} \theta_i I(country_i) + \phi_1 Growth_{t-1} \\ + \phi_2 POPGROWTH_t + \phi_3 GOVSIZE_t + U_t \end{aligned} \tag{20}$$

where $GDPCAPITAGROWTH_t$ is GDP per capita growth at year t and other variables the same as from equation 9 and 10.

Using the full data set, the estimated threshold from equation 19 is 108% which is very similar to 109% calculated from equation 9. The coefficient corresponds to $I(d_{t-1} > 108\%)$ is -0.734 which is also very close to -0.876 estimated from equation 9 (Table 2.6). The result is significant at 5% level. However, the smaller coefficient means that given other factors the same, the adverse effect of debt on growth of GDP per capita is smaller than on output growth. The estimated threshold from equation 20 is 115% which is the same as the threshold estimation of 115% from equation 10 (Table 2.7). Both difference may be explained by measurement error.

Using the data before the financial crisis, the estimated thresholds from equation 19 is 71% which is the same as my estimates from equation 9. The coefficient corresponds to $I(d_{t-1} > 108\%)$ is -0.985 which is also very close to -1.27 estimated from equation 9 (Table 2.6). Similar as using the full data, the smaller coefficient means that given other factors the same, the adverse effect of debt on growth of GDP per capita is smaller than on output growth. The threshold estimation from equation 20 is 77% which is very close to 76% estimated by equation 10 (Table 2.7). Again, the difference may be explained by measurement error. Using growth of GDP per capita as dependent variable, I observe a decrease in threshold value using data before the financial crisis.

Using data after the financial crisis, equation 19 fail to give an estimated threshold value because all coefficient correspond to $I(d_{t-1} > d^*)$ for integer valued d^* between 30 and 150 are all positive. However, similar as the result from section 4, coefficient does have a creasing trend as threshold value increase. Using threshold value found in section 4 by equation 9, the coefficient of $I(d_{t-1} > 109\%)$ is 0.287 (Table 2.6). Equation 20, similar to equation 10 fail to give a practical estimation of threshold (Table 2.7). Therefore, using growth of GDP per capita as dependent variable, I can draw the same conclusion that after the financial crisis countries do seem more tolerance to debt.

Overall, using growth of GDP per capita as dependent variable, I got very almost identical result as in section 4. Therefore, my results are robust and should be free from calculation error.

6 Conclusion and Further Research

6.1 Conclusion

Debt threshold a topic subject to heated academic debate in the last decade. What is the effect of public debt accumulation on economic growth is an central question to policy maker when designing the optimal fiscal policy. This topic gains increasing attention in the aftermath of financial crisis in 2008. This paper aims to answer the question of the relationship between government debt and economic growth and if financial crisis has an impact on the relationship, by using a large cross sectional data set.

I developed two estimation methods to capture any threshold effects while controlling for endogeneity. Then I conducted formal statistical analysis of debt threshold effect on output growth by applying the two estimation methods on a panel data which spans 30 years and consists 30 European countries, as well as on two subgroups classified by credit rating. To find the impact of financial crisis on the debt threshold effect, I splitted the panel data into two subgroups, data before and after financial crisis, and performed statistical analysis using the two estimation methods on.

I found a inverted U shape relationship between debt and growth in some cases. Statistically significant threshold effect was found using both estimation methods on the full data sets. However, my estimates are not identical (109% and 115%). The short-term impact of debt on GDP growth become negative and less significant once the public debt to GDP ratio reaches 109% or 115%. In addition, the threshold effect is heterogeneous by crediting rating cohort. Countries with low credit rating are more vulnerable to debt (estimated threshold of 60%) compare to countries with high credit rating (estimated threshold of 115%). However, this result is less statistically significant due to data limitation. Furthermore, financial crisis does have considerable impact on threshold effect. Countries become substantially more tolerance to debt. Before the financial crisis, estimated thresholds from the two estimation methods are 71% and 76%, whereas after the financial crisis, no thresholds were found. This means that after the financial crisis, there does not exists a debt to GDP ratio for which any further accumulation in debt will have a reversal effect on growth.

My results show that the positive short term economic stimulus from additional debt decreases severely when the initial debt level is high, and can become negative as debt increases. The adverse effect suggests that when debt to GDP ratio is extremely high, reducing it would be beneficial to economic growth. On the other hand, if the initial debt is low, increasing debt would have a positive effect on growth in the short run, which in line with conventional Keynesian multipliers effect. Hence, my result supports the idea that increasing debt stimulates the economy only when the initial debt level is below certain threshold. However, one should be caution when putting these results in practice because every country is unique in the sense that state of affairs vary across nation.

6.2 Further Research

Further research can be done to investigate mid-term and long term effect debt accumulation and growth. Whether similar threshold effect exist in Asian and North and South Americas also worth study. The channel which increase in debt effects growth should also be studied as policy maker than introduce policies through these channels to counter the negative effect of debt on growth. The effect of foreign debt on growth is also an intriguing topic.

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Appendix 1

Table 1.1: Data Description and Source

Variable Abbreviation	Variable Name	Unit	Source
GROWTH	real GDP growth rate	%	AMECO
GDPCAPITAGROWTH	GDP per capita growth	%	AMECO
d	debt to GDP ratio	%	AMECO
POPGROWTH	population growth	%	AMECO
GOVSIZE	total government expenditure to GDP ratio	%	AMECO
INT^l	real long term interest rate	%	AMECO
INT^s	real short term interest rate	%	AMECO
Inflation	Inflation	%	AMECO

Note: Countries included are Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and United Kingdom.

Table 1.2: Data Summary

Variable Abbreviation	Mean (%)	Standard Deviation
GROWTH	2.40	3.51
GDPCAPITAGROWTH	2.04	3.55
d	60.29	31.91
POPGROWTH	0.354	0.806
GOVSIZE	45.21	6.58
INT^l	4.39	2.35
INT^s	0.540	2.74
Inflation	0.959	0.130

Note: Countries included are the same as in Table 1.1.

Appendix 2

Table 2.1: Regression Result from Equation 9

Variable	Complete Data	Before Financial Crisis	After Financial Crisis	Low Credit Rating	High Credit Rating
$I(d_{t-1} > d^*)$	-0.8758^{**}	-1.2692^{***}	0.3747	-0.04287	-0.7648^{*}
	(0.4092)	(0.3663)	(0.4137)	(0.1921)	(0.4675)
$GROWTH_{t-1}$	0.1547^{***}	0.3526^{***}	0.05621^{***}	0.1837	0.04074^{*}
	(0.04987)	(0.07286)	(0.05810)	(0.1288)	(0.03062)
$POPGROWTH_t$	0.6593^{*}	0.2294	0.3355^{*}	0.6784^{*}	0.1297^{*}
	(0.4038)	(0.3182)	(0.2197)	(0.5927)	(0.2734)
$GOVSIZE_t$	-0.4871^{***}	-0.2381^{***}	-0.3639^{***}	-0.4583^{***}	-0.2621^{***}
	(0.04350)	(0.03598)	(0.06616)	(0.1540)	(0.05903)
$I(POST2008)$	-3.0983^{***}			-3.2927^{***}	-1.9511^{***}
	(0.2974)			(1.0385)	(0.3323)
$I(POST2008) * I(d_{t-1} > d^*)$	-1.5403^{*}			-1.5597	-1.4233^{*}
	(0.9922)			(1.4777)	(0.8975)
Country Dummy	Included (30)	Included (30)	Included (30)	Included (5)	Included (9)
Debt Threshold	109%	71%	NA	60%	115%

Note: The dependent variable is GDP Growth. The abbreviations for the explanatory variables are explained in Table 1.1, Appendix 1. Countries included in complete data, before financial crisis, after financial crisis are Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and United Kingdom. Countries included in low credit rating are Bulgaria, Croatia, Cyprus, Hungary, Italy, and Romania. Countries included in high credit rating are Austria, Germany, Sweden, Netherlands, Norway, Luxembourg, Belgium, Denmark, Finland, and France. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is the indicator on debt to GDP ratio above threshold.

Table 2.2: Regression Result from Equation 10

Variable	Complete Data	Before Financial Crisis	After Financial Crisis	Low Credit Rating	High Credit Rating
d_{t-1}	0.1267**	0.1023***	−0.03048	0.06125	0.1020*
	(0.05711)	(0.03950)	(0.03536)	(0.8329)	(0.04432)
${d_{t-1}}^2$	0.0005461*	0.0006730*	0.9544***	0.006838	−0.0004286
	(0.0003988)	(0.0003787)	(0.2467)	(0.01034)	(0.002882)
$GROWTH_{t-1}$	0.1398***	0.1489***	0.02017	0.1459	0.07033
	(0.04122)	(0.05949)	(0.05786)	(0.1065)	(0.07610)
$POPGROWTH_t$	0.9736**	0.8672*	1.1762*	0.6594	0.1806
	(0.4946)	(0.4478)	(0.6540)	(0.6751)	(0.2090)
$GOVSIZE_t$	−0.3318***	−0.3805***	−0.3699***	−0.4459***	−0.3070***
	(0.04124)	(0.06778)	(0.06641)	(0.1335)	(0.06093)
$I(POST2008)$	−4.542***			−4.6208***	4.0658***
	(0.5862)			(1.6624)	(1.1471)
$I(POST2008) * d_{t-1})$	0.04621***			0.5231	0.06677
	(0.009401)			(1.0397)	(0.4969)
Country Dummy	Included (30)	Included (30)	Included (30)	Included (5)	Included (9)
Debt Threshold	115%	76%	NA	NA	119%

Note: The dependent variable is GDP Growth. The abbreviations for the explanatory variables are explained in Table 1.1, Appendix 1. Countries included in complete data, before financial crisis, after financial crisis are Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and United Kingdom. Countries included in low credit rating are Bulgaria, Croatia, Cyprus, Hungary, Italy, and Romania. Countries included in high credit rating are Austria, Germany, Sweden, Netherlands, Norway, Luxembourg, Belgium, Denmark, Finland, and France. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is one year lagged debt to GDP ratio and squared one year lagged debt to GDP ratio.

Table 2.3: Long Term Interest Rate as Dependent Variable

Variable	Complete Data	Before Financial Crisis	After Financial Crisis
$I(d_{t-1} > d^*)$	0.01002	0.0273*	0.2293
	(0.2035)	(0.01992)	(0.3288)
$GROWTH_{t-1}$	0.1547***	0.0007773	0.03498
	(0.04987)	(0.04674)	(0.04063)
${INT_{t-1}}^s$	0.1853***	0.2746***	0.1492***
	(0.03229)	(0.03222)	(0.05082)
$GOVSIZE_t$	0.1960***	0.1071***	0.2614***
	(0.02811)	(0.03536)	(0.04220)
$I(POST2008)$	−1.6779***		
	(0.1876)		
Country Dummy	Included (30)	Included (30)	Included (30)
Debt Threshold	103%	33%	130%

Note: The dependent variable is long term interest rate. The abbreviations for the explanatory variables are explained in Table 1.1, Appendix 1. Countries included in complete data, before financial crisis, after financial crisis are the same as in table 2.2. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is indicator on debt to GDP ratio over threshold.

Table 2.4: Regression Result Without Influential Data Using Equation 9

Variable	Model1	Model2	Model3	Model4	Model5
$I(d_{t-1} > d^*)$	−0.3101*	−0.322*	−0.2760	−0.3742	−0.1063
	(0.1835)	(0.1886)	(0.1980)	(0.2771)	(0.3876)
$GROWTH_{t-1}$	0.1497***	0.1526***	0.1616***	0.1000**	0.1460***
	(0.03874)	(0.04252)	(0.04390)	(0.04371)	(0.04838)
$POPGROWTH_t$	0.6510*	0.5525	0.09982	0.2215	0.05687
	(0.4122)	(0.6060)	(0.3062)	(0.3090)	(0.3294)
$GOVSIZE_t$	−0.4878***	−0.2927***	−0.2950***	−0.3040***	−0.2756***
	(0.04189)	(0.04056)	(0.04084)	(0.04337)	(0.04506)
$I(POST2008)$	−5.6732***	−4.8313***	−5.6433***	−6.4728***	−5.4342***
	(0.6901)	(0.6777)	(0.6991)	(0.7789)	(0.7570)
$I(POST2008) * I(d_{t-1} > d^*)$	−3.2558**	−3.2398**	−3.2012***	−2.7894***	−3.1401***
	(1.7987)	(1.3801)	(1.1851)	(0.9745)	(0.9438)
Country Dummy	Included (29)	Included (29)	Included (27)	Included (27)	Included (24)
Debt Threshold	111%	109%	128%	105%	106%

Note: The dependent variable is GDP growth. Explanatory variables are the same as in table 2.1. Model 1 excludes Estonia. Model 2 excludes Greece. Model 3 excludes Estonia, Luxembourg, and Bulgaria. Model 4 excludes Greece, Italy, and Belgium. Model 5 excludes Estonia, Luxembourg Bulgaria, Greece, Italy, and Belgium. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is indicator on debt to GDP ratio over threshold.

Table 2.5: Regression Result Without Influential Data Using Equation 10

Variable	Model1	Model2	Model3	Model4	Model5
d_{t-1}	0.06053**	0.08061*	0.06273*	0.09009	0.03440
	(0.03008)	(0.04773)	(0.04521)	(0.07525)	(0.05227)
${d_{t-1}}^2$	0.0002587*	0.0002757*	0.0002681	0.0004058	−0.0001592
	(0.0001536)	(0.0001688)	(0.0001792)	(0.0003111)	(0.001555)
$GROWTH_{t-1}$	0.1536***	0.1655***	0.1177****	0.1201***	0.1351***
	(0.04468)	(0.04453)	(0.04378)	(0.04371)	(0.04874)
$POPGROWTH_t$	0.1497	0.3161	0.06979	0.2215	0.2433
	(0.1049)	(0.3164)	(0.1066)	(0.3090)	(0.3343)
$GOVSIZE_t$	−0.3160***	−0.3040***	0.3993***	−0.3271***	−0.2658***
	(0.04182)	(0.04125)	(0.04380)	(0.04237)	(0.04484)
$I(POST2008)$	−5.0439***	−4.8130***	4.4910***	4.0728***	−3.3903***
	(0.6660)	(0.7880)	(0.7871)	(0.7489)	(0.9786)
$I(POST2008) * d_{t-1})$	0.04303***	0.04191***	0.08859***	0.03539***	0.04191***
	(0.01456)	(0.01593)	(0.01544)	(0.01462)	(0.01859)
Country Dummy	Included (29)	Included (29)	Included (27)	Included (27)	Included (24)
Debt Threshold	117%	115%	117%	111%	108%

Note: The dependent variable is GDP growth. Explanatory variables are the same as in table 2.2. Model 1 excludes Estonia. Model 2 excludes Greece. Model 3 excludes Estonia, Luxembourg, and Bulgaria. Model 4 excludes Greece, Italy, and Belgium. Model 5 excludes Estonia, Luxembourg Bulgaria, Greece, Italy, and Belgium. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is one year lagged debt to GDP ratio and squared one year lagged debt to GDP ratio.

Table 2.6 Growth of GDP per Capita as Dependent Variable Using Equation 9

Variable	Complete Data	Before Financial Crisis	After Financial Crisis
$I(d_{t-1} > d^*)$	−0.7343**	−0.9851***	0.2870
	(0.4521)	(0.3592)	(0.3777)
$GROWTH_{t-1}$	0.1687***	0.3626***	0.05553**
	(0.04787)	(0.07115)	(0.02615)
$GOVSIZE_t$	−0.4933***	−0.2445***	−0.3546***
	(0.04272)	(0.03220)	(0.07112)
$I(POST2008)$	−3.8418***		
	(0.3788)		
$I(POST2008) * I(d_{t-1} > d^*)$	−1.7010*		
	(1.0035)		
Country Dummy	Included (30)	Included (30)	Included (30)
Debt Threshold	108%	71%	NA

Note: The dependent variable is GDP per capita growth. Explanatory variables are the same as in table 2.1. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is indicator on debt to GDP ratio over threshold.

Table 2.7 Growth of GDP per Capita as Dependent Variable Using Equation 10

Variable	**Complete Data**	**Before Financial Crisis**	**After Financial Crisis**
d_{t-1}	0.1335^{**}	0.1243^{***}	-0.03548
	(0.03635)	(0.02424)	(0.03920)
${d_{t-1}}^2$	0.0006181^{*}	0.0008071^{*}	0.8795^{***}
	(0.0003887)	(0.0004928)	(0.1962)
$GROWTH_{t-1}$	0.1599^{***}	0.3178^{***}	0.04729
	(0.05643)	(0.07985)	(0.04534)
$GOVSIZE_t$	-0.4398^{***}	-0.2019^{***}	-0.3911^{***}
	(0.05329)	(0.03918)	(0.06774)
$I(POST2008)$	-4.3837^{***}		
	(0.4651)		
$I(POST2008) * d_{t-1}$	0.03211^{***}		
	(0.01059)		
Country Dummy	Included (30)	Included (30)	Included (30)
Debt Threshold	115%	77%	NA

Note: The dependent variable is GDP per capita growth. Explanatory variables are the same as in table 2.2. The table shows the estimated coefficients, standard errors (SE) which are in parentheses and their significance level (*10%; **5%, ***1%). The main variable of interest is one year lagged debt to GDP ratio and squared one year lagged debt to GDP ratio.

Appendix 3

Figure 3.1 Average Debt to GDP Ratio by Year

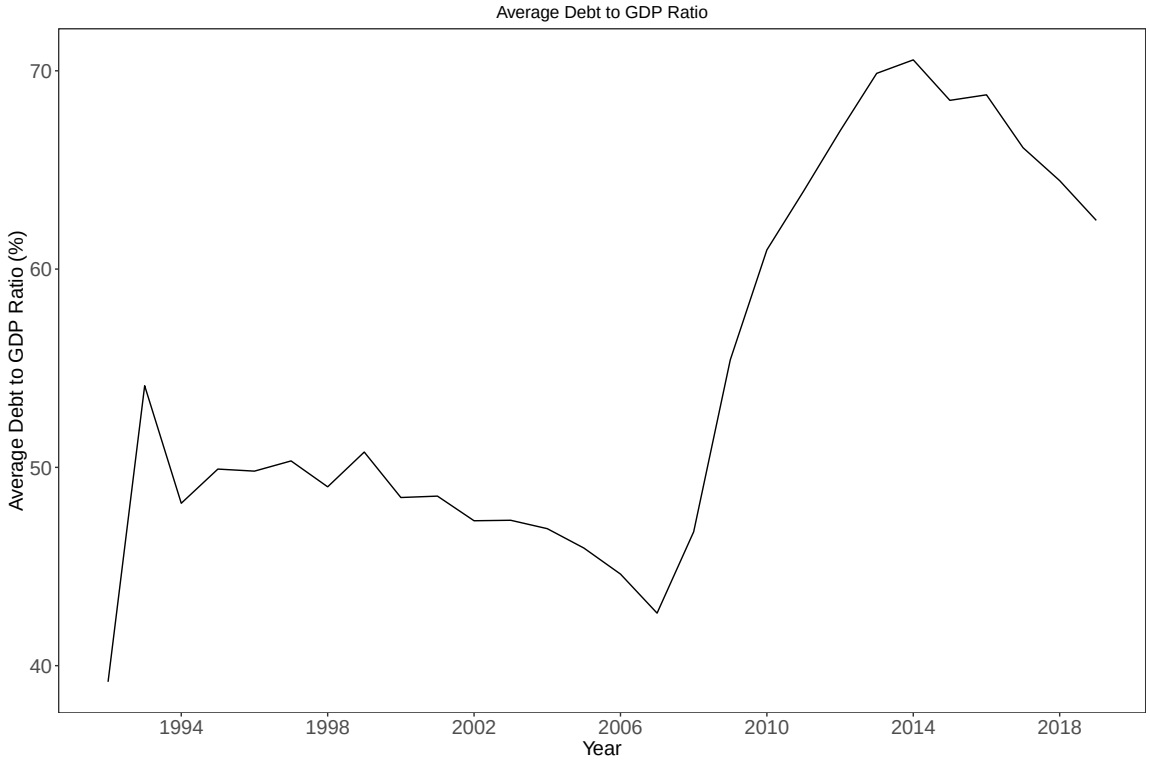


Figure 3.2 Average Economic Growth by Year

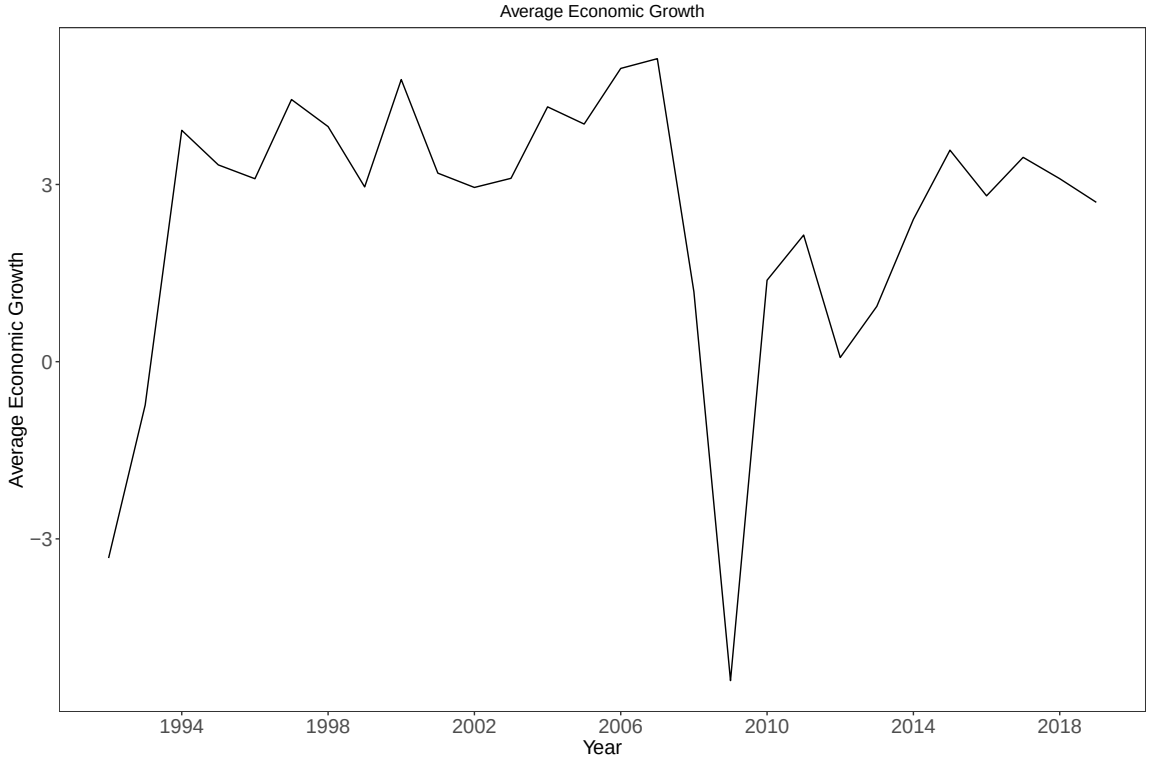


Figure 3.3 Average Economic Growth by Year

Figure 3.4 Coefficient On Debt Threshold (Full Data)

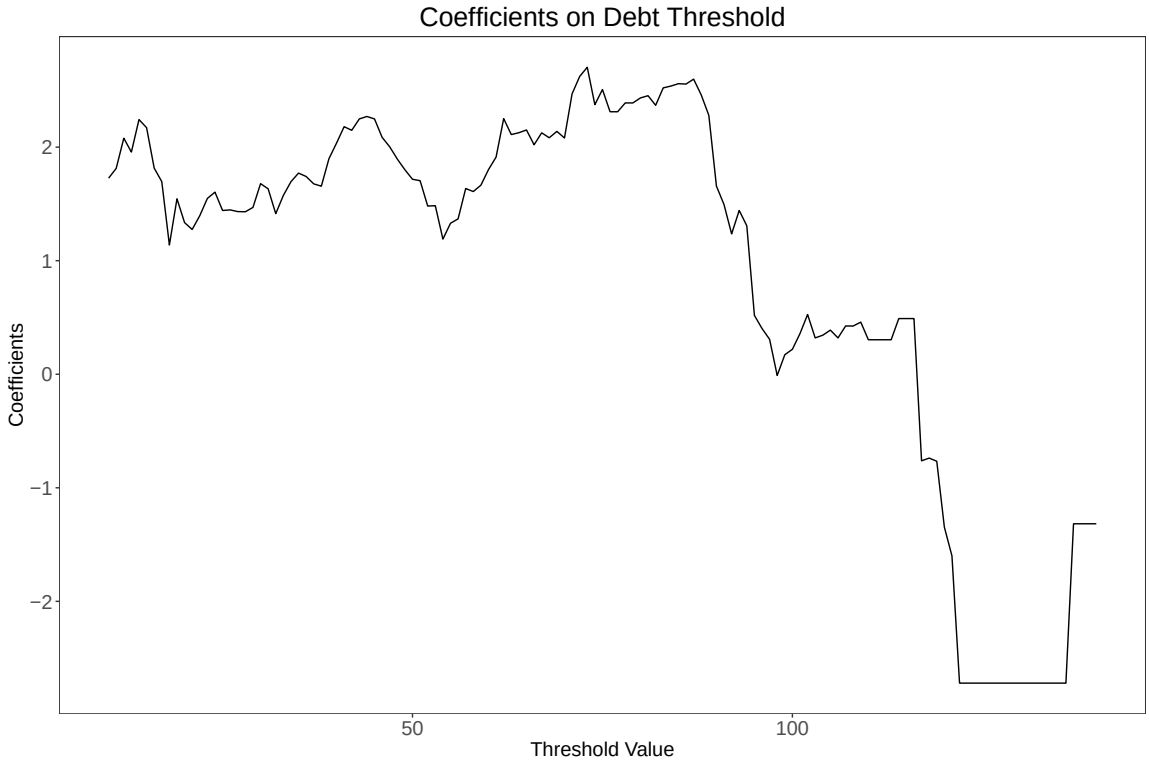


Figure 3.5 Coefficient On Debt Threshold (Before Financial Crisis)

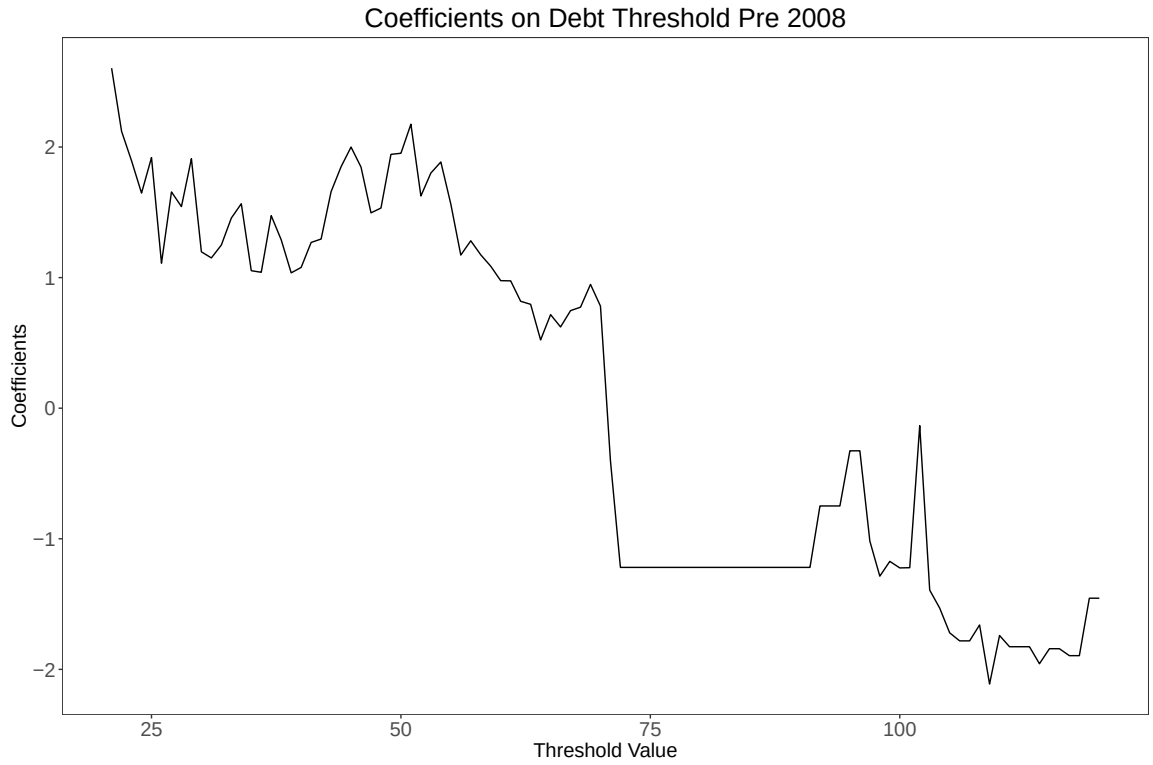


Figure 3.6 Coefficient On Debt Threshold (After Financial Crisis)

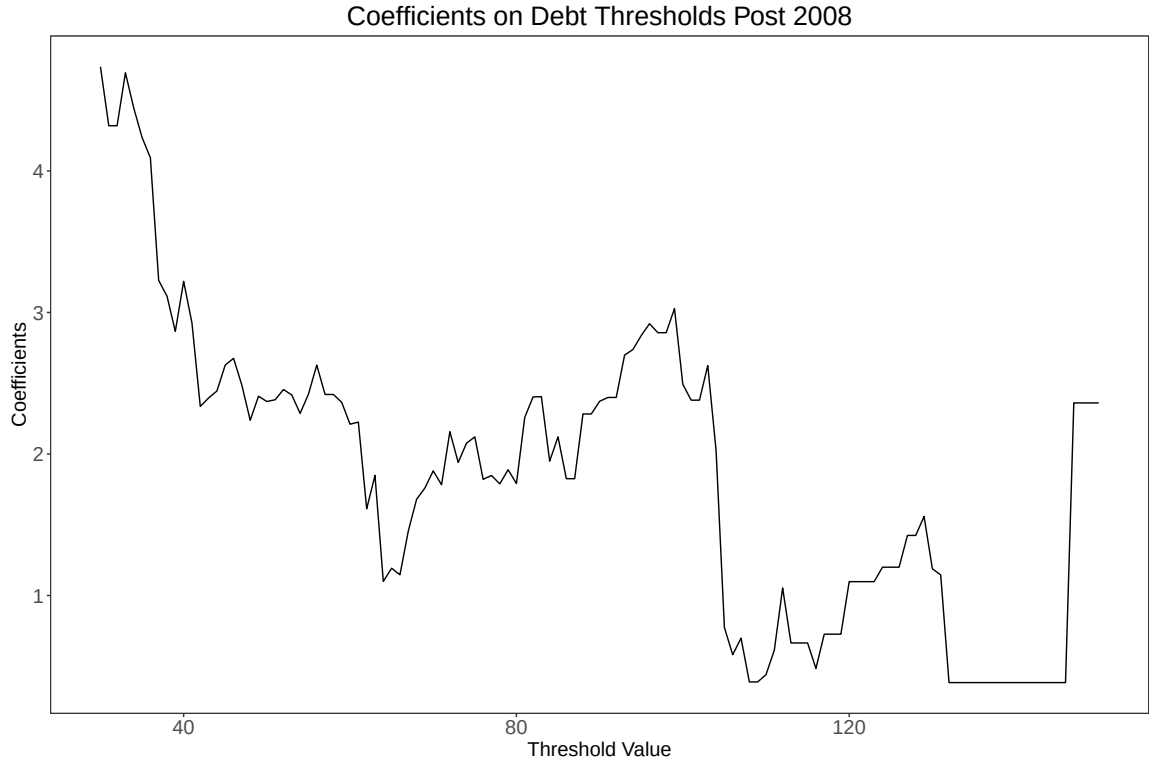


Figure 3.7 Coefficient On Debt Threshold (Low Credit Rating)

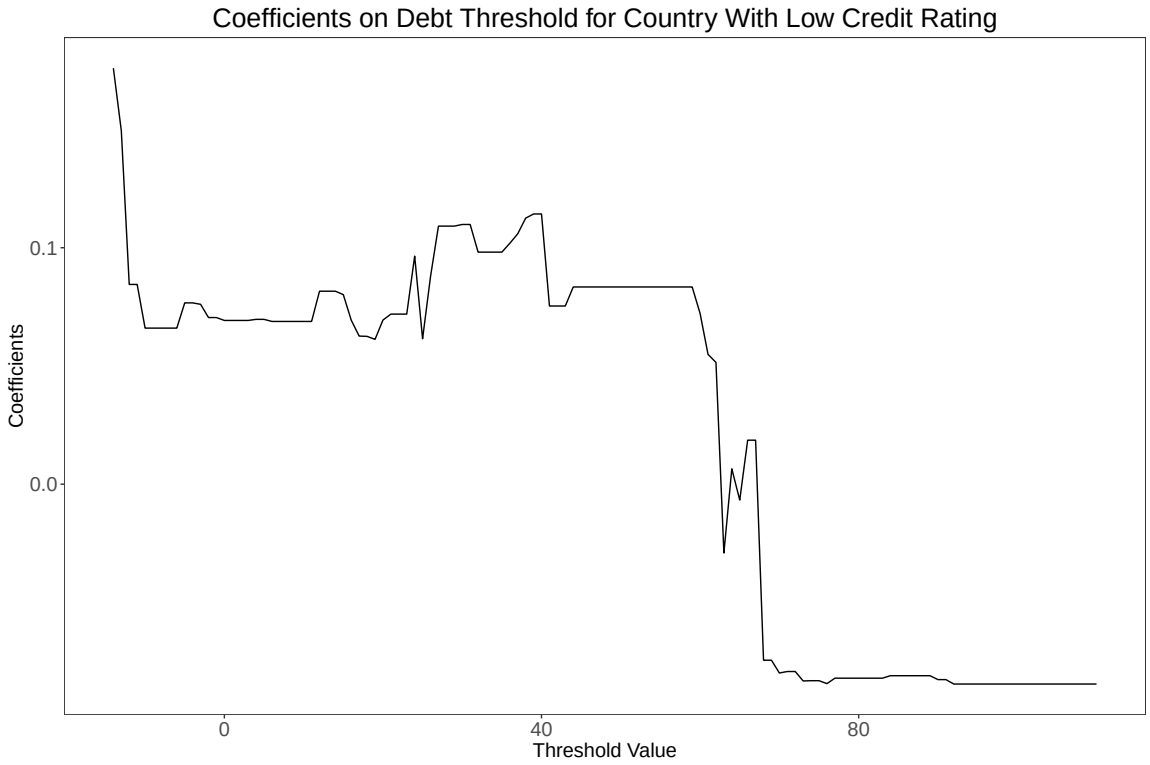


Figure 3.8 Coefficient On Debt Threshold (High Credit Rating)

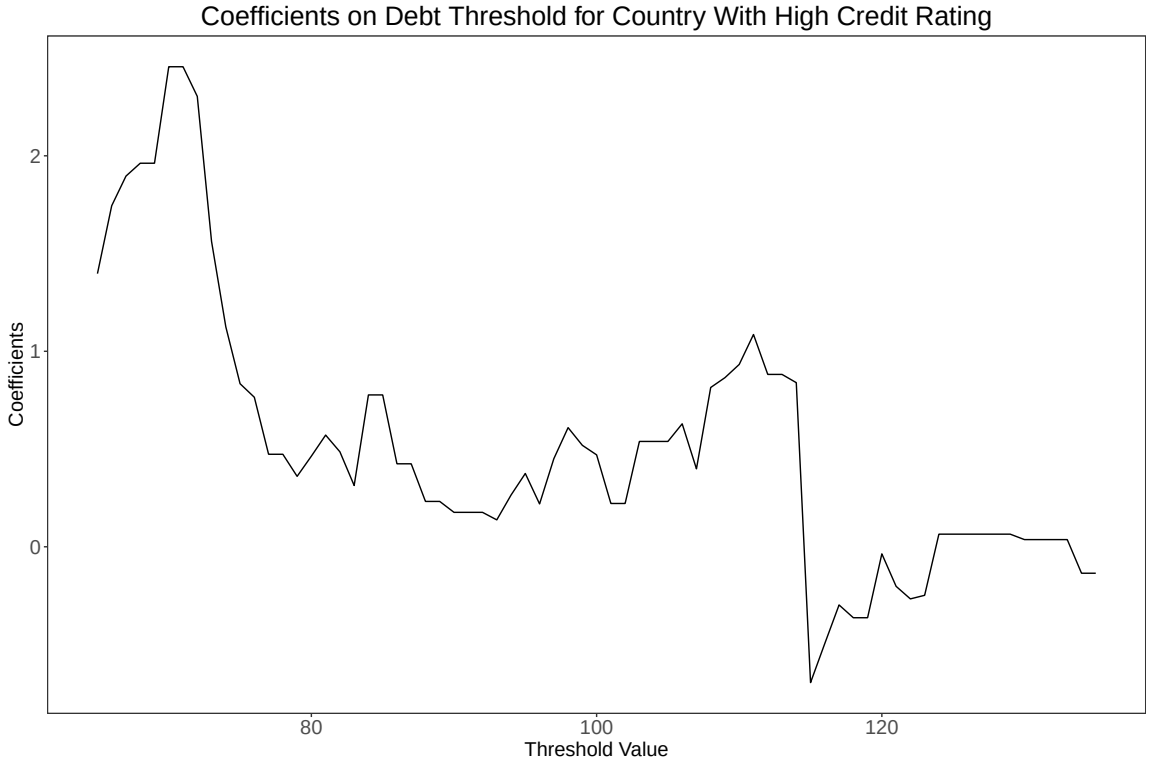


Figure 3.9 Coefficient On Debt Threshold (Full data, Long Term Interest Rate as Dependent Variable)

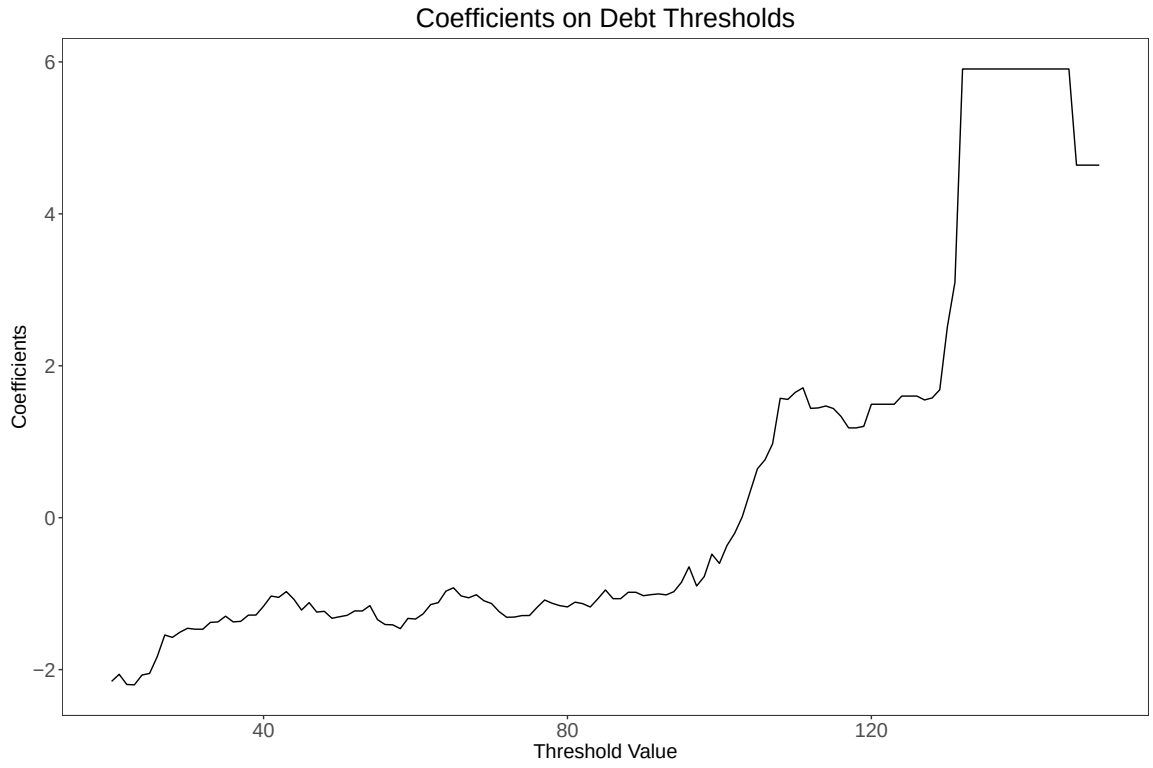


Figure 3.10 Coefficient On Debt Threshold (Before Financial Crisis, Long Term Interest Rate as Dependent Variable)

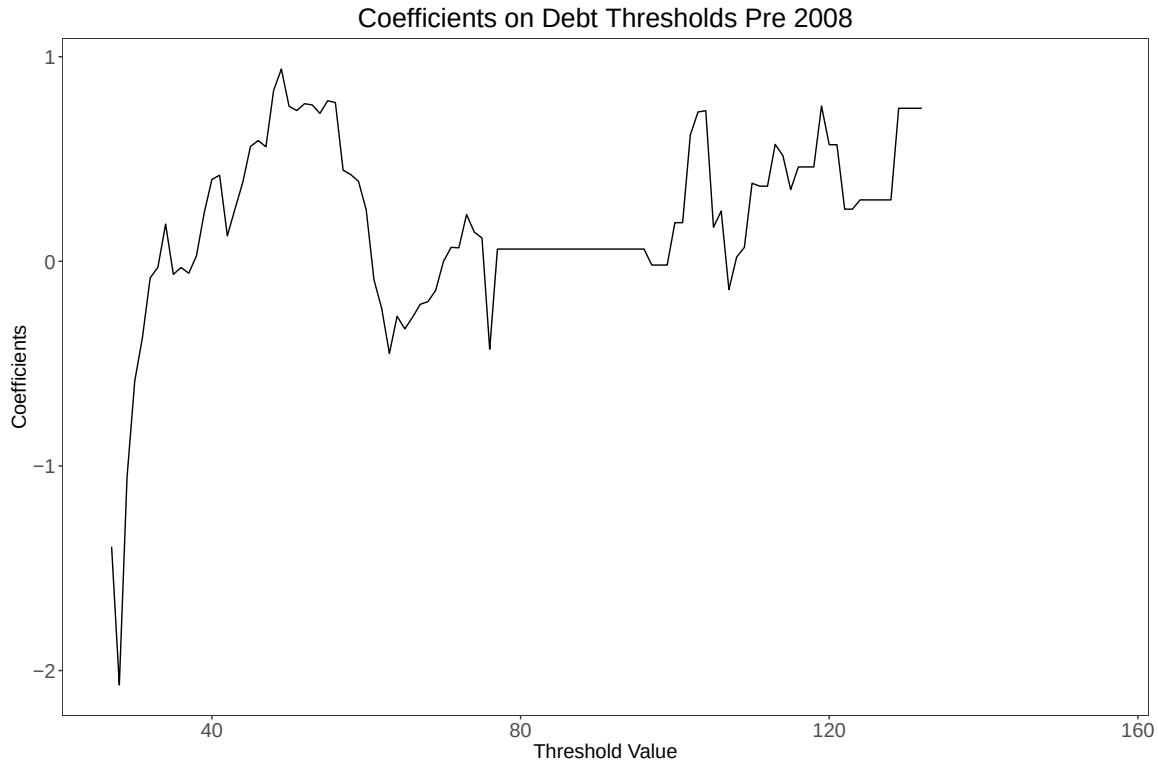
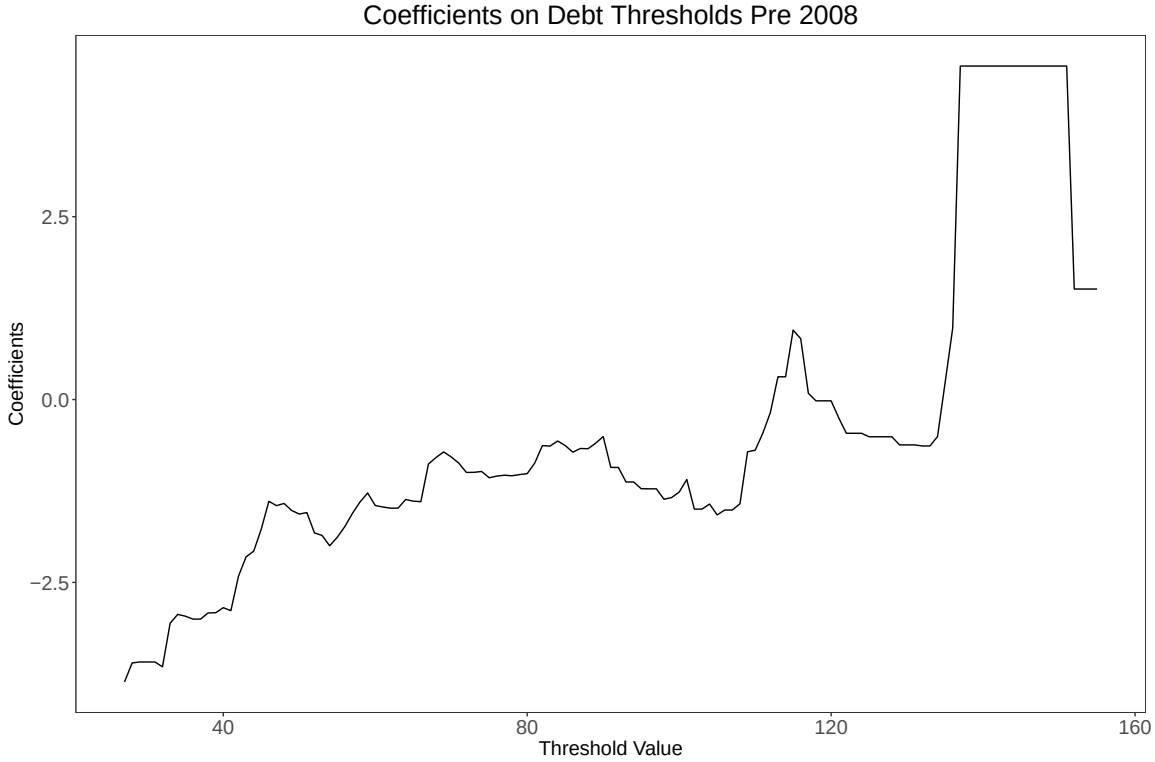


Figure 3.11 Coefficient On Debt Threshold (After Financial Crisis, Long Term Interest Rate as Dependent Variable)



Figures corresponds to estimations in section 5.2 and 5.3 are very similar to figure 3.4, figure 3.5 and figure 3.6. One can refer to these figures for estimations in section 5.2 and 5.3