THEORY FIELD EXAM
WINTER 2011

INSTRUCTIONS

You must complete two of the three areas (the areas being (I) contract theory, (II) game theory, and (III) psychology & economics). Be sure to indicate clearly what you are answering. Try to be succinct in your answers, especially with respect to those questions that ask for discussion. Write legibly.

AREA I: CONTRACT THEORY QUESTION (ECON 206)

Instructions: Answer all parts.

Consider a risk-neutral principal and a risk-averse agent. The game between them is a standard hidden-action agency game (*e.g.,* as considered by Holmstrom, 1979, or Grossman & Hart, 1983). Let $\mathcal{A}$ be the set of possible actions. Let $\mathcal{X} \subseteq \mathbb{R}$ be the set of possible outcomes. Let $\boldsymbol{\pi}(a)$ denote the density over elements of $\mathcal{X}$ if the agent chooses $a \in \mathcal{A}$. Assume the realized outcome, x, is verifiable. Let $\underline{U}$ denote the agent's reservation utility. Assume the agent's utility is $u(y) - c(a)$, where y is income, $c : \mathcal{A} \to \mathbb{R}_+$, and $u : (\underline{y}, \infty) \to \mathbb{R}$, $\underline{y}$ a constant (possibly $-\infty$). Assume that $u(\cdot)$ is strictly increasing with $\lim_{y \downarrow \underline{y}} u(y) = -\infty$ and $\lim_{y \uparrow \infty} u(y) = \infty$. Assume the principal's utility is $x - y$. Under the *hidden-action* game, let $\mathcal{A}^I$ denote the set of *implementable* actions.

(a) Suppose that $\mathcal{A}$ and $\mathcal{X}$ are finite sets. Fix an $a \in \mathcal{A}$. Give necessary and sufficient conditions for that a to be an element of $\mathcal{A}^I$.

Consider the following variation of the above game. Suppose that between the time the agent chooses his action and the time an outcome is realized, the principal observes—but cannot verify—the agent's action. Moreover, in this time interval, the principal can offer the agent a new contract (*i.e.,* renegotiate) on a take-it-or-leave-it (TIOLI) basis.

(b) Let $s : \mathcal{X} \to [\underline{y}, \infty)$ denote the original contract (*i.e.,* the one in place at the time the agent chooses his action). Suppose the principal observes that the agent chose $\hat{a}$. What contract would the principal offer the agent in renegotiation?

(c) Suppose the original contract is determined by the principal's making a TIOLI offer to the agent. Prove that the set $\mathcal{A}^I$ is the same whether or not there is subsequent renegotiation of the sort described above (*i.e.,* prove that $a \in \mathcal{A}^I$ when renegotiation is impossible if and only if $a \in \mathcal{A}^I$ when renegotiation is possible).

(d) Same assumptions as the previous question. If $a \in \mathcal{A}^I$, what would be the principal's expected cost (*i.e.,* expected payment to the agent) in the equilibrium of the subgame in which she seeks to implement a and renegotiation of the sort described above is possible.

Consider the following variation of the renegotiation game. It is uncertain at the time of initial contracting and when the agent takes his action whether the principal will or won't observe the agent's action in the interval between when the agent takes his action and the outcome is realized. Let $\psi \in (0, 1)$ denote the probability that the principal does observe the agent's action in that interval. Whether or not the principal observes the agent's action, she can propose a new contract at the renegotiation stage on a TIOLI basis. Assume there is an $\underline{a} \in \mathcal{A}$ such that $c(\underline{a}) < c(a)$ for all $a \in \mathcal{A}$ such that $a \neq \underline{a}$. Further assume that $\mathcal{A}$ and $\mathcal{X}$ are finite and, for all $x \in \mathcal{X}$ and all $a \in \mathcal{A}$, the probability of realizing x given a is positive (*i.e.,* if $\pi(x|a)$ is the probability of x given a, then $\pi(x|a) > 0$ $\forall (x, a) \in \mathcal{X} \times \mathcal{A}$).

(e) Consider play on the equilibrium path if the principal seeks to implement $a \in \mathcal{A}^I$, $a \neq \underline{a}$, in the game in which she is certain to observe the agent's action (*i.e.,* $\psi = 1$, as in the earlier questions). Let $s(\cdot)$ be the initial contract offered on that path. Prove that if the principal offered that same $s(\cdot)$ in the variation of the game in which $\psi < 1$, then the agent will *not* choose to play a. Interpret your result.

(f) Suppose that $\mathcal{A} = \{0, 1\}$ and $c(a) = a$. Suppose that, in renegotiation, the principal simply offers a payment of her, then, choosing to the agent in exchange for his relinquishing his claims under the original contract. What is the equilibrium of this game (recall, now, $\psi \in (0, 1)$)?

(g) Offering just a single payment is actually not the optimal thing for the principal to do when she doesn't observe the agent's action. What should she offer in this case? (Note: You don't have to solve anything here, just state what she should do.)

Area II: Game Theory Questions (Econ 209A)

Instructions: Answer both questions. For the second question, answer all parts.

Question 1: Let **B** be the set of all convex, compact, and comprehensive bargaining problems $\langle S, d \rangle$ in $\mathbb{R}^2_+$ with nonempty intersection with $\mathbb{R}^2_{++}$ (a set X in $\mathbb{R}^2_+$ is *comprehensive* if $\mathbf{x} \in X$ and $y_1 \leq x_1$ and $y_2 \leq x_2$ then $\mathbf{y} \in X$).

Show that the Kalai-Smorodinsky solution

$$f^{KS}(S, d) = \{\frac{s_1}{\bar{s}_1} = \frac{s_2}{\bar{s}_2} : s \in S\} \cap WPO(S)\,,$$

where $\bar{s}_i$ be the maximum utility player i gets in $s \in S$, is the unique bargaining solution satisfying *SYM*, *WPO*, *INV*, and individual monotonicity (INM):

(i) For any $\langle S, d \rangle$ and $\langle T, d \rangle$ with $S \subset T$ and $\bar{s}_i = \bar{t}_i$ for $i = 1, 2$, we have

$$f_i(S, d) \leq f_i(T, d)$$

for $i = 1, 2$.

(ii) For any $\langle S, d \rangle$ and $\langle T, d \rangle$ with $S \subset T$ and $\bar{s}_i = \bar{t}_i$ for i, we have

$$f_j(S, d) \leq f_j(T, d)$$

for $j \neq i$.

Question 2: Consider a payoff symmetric game

$$G = \langle \{1, 2\}, (A, A), (u_i) \rangle$$

where $u_1(a) = u_2(a')$ when a' is obtained from a by exchanging a_1 and a_2. Prove or give a counter-example:

(a) The game possesses a symmetric Nash equilibrium, that is a Nash equilibrium in which both players choose the same strategy (pure or mixed).

(b) If a^* is an evolutionary stable (pure) strategy then it weakly dominates every other strategy?

(c) If α^* is an evolutionary stable (mixed) strategy and there exist a and $a' \neq a$ such that $\alpha^*(a) > 0$ and $\alpha^*(a') > 0$ then (a, a) is not a pure strategy NE.

(d) The payoff of each player is zero in every Nash equilibrium of a symmetric zero-sum game.

Area III: Psychology and Economics Question (Econ 219A)

Instructions: Answer all parts.

Ben must do a task exactly once in his infinite lifetime. The day he does it his instantaneous utility is $u_t = -1$. The day immediately after he does the task it is $u_t = 2$. On all other days it is $u_t = 0$. Ben cannot commit ahead of time as to when he will do it.

Ben has standard present-biased preferences given by

$$U^t \equiv u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau ,$$

with $\beta \in (0, 1]$, and $\delta \in (0, 1)$. These are standard present-biased preferences. Ben might be either sophisticated ($\widehat{\beta} = \beta$) or naive ($\widehat{\beta} = 1$).

For all questions below, don't worry about specifying behavior for any knife-edge values of parameters that make a person indifferent among choices in some contingencies in ways that won't arise generically for parameter values. Also, many questions ask for ranges of parameters where some type of equilibrium is possible; do not spend time specifying the actual equilibria—just answer the ranges of parameters as asked. Please simply but clearly specify the relevant maximization problems in each part to get partial credit in case of algebraic or arithmetic difficulties.

(a) Suppose that $\beta = 1$, so that Ben is time consistent. For what values of $\delta < 1$ will Ben do the task the first period?

(b) For what values might time-consistent Ben do the task for sure, but not immediately?

(c) Suppose that $\beta < 1$, $\delta < 1$, and $\widehat{\beta} = 1$. That is, Ben is present-biased and naive. For what values of β and δ will Ben for sure do the task immediately?

(d) For what values of β and δ will naive Ben not do the task right away, but will do the task in some later period?

(e) Suppose that $\beta < 1$, $\delta < 1$, and $\widehat{\beta} = \beta$. That is, Ben is present-biased and sophisticated. For what values of β and δ does there exist a "perception-perfect equilibrium" (i.e., the way we always make predictions for sophisticates) in which Ben does the task immediately with probability 1?

(f) For what values of β and δ does there exist a "perception-perfect equilibrium" in which sophisticated Ben for sure *never* does the task?

Cendri is considering the same task as Ben, and is similarly unable to commit. But she does not have any self-control problem: each period she seeks to maximize her continuation utility $U^t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u_\tau$, with $\delta \in (0, 1)$. And Cendri has preferences that are related but somewhat different preferences than Ben. Her "consumption utility" is just like Ben's. The day she does it her instantaneous consumption utility is $m_t = -1$. The day immediately after she does the task it is $m_t = 2$. On all other days it is $m_t = 0$. But Cendri also has anticipatory utility that, each period, is proportional to her expected total future consumption utility. Formally, in period t, if she anticipates a future flow of consumption utilities of $m_{t+1}, m_{t+2}, ...$ then her "anticipatory utility" in period t is $a_t \equiv \varphi \sum_{\tau=t}^{\infty} \delta^{\tau-t} m_\tau$, with the same δ as above, and $\varphi > 0$ is a preference parameter. (If Cendri is uncertain about the profile of future consumption utilities, her anticipatory utility is proportional to the expected value of this sum.) Cendri's utility in period t is then simply $m_t + a_t$.

Cendri is completely rational and plays a "personal equlibrium" in the usual sense (in which her beliefs about her own behavior, payoffs, etc. must always be rational, but she must maximize her utility at each continuation point). Note: there may be more than one reasonable interpretation/variant of "personal equilibrium" consistent with various material you have been exposed to; do whatever seems natural to you and be clear how you reached your conclusions.

(g) For what values of δ and φ does there exist a personal equilibrium in which Cendri for sure *never* does the task? Succinctly make clear how you reached this conclusion.

(h) For what values of δ and φ does there exist a personal equilibrium in which Cendri for sure does the task *immediately*? Succinctly make clear how you reached your conclusions.

(i) Time permitting (do not sacrifice other parts of this exam to address this), consider the question of whether there exists any values of δ and φ in which Cendri might get a higher lifetime utility if she were able to commit to a strategy.