Theory Field Exam
Summer 2011

Instructions

You must complete two of the four areas (the areas being (I) contract theory, (II) game theory A, (III) game theory B, and (IV) psychology & economics). Be sure to indicate clearly what you are answering. Try to be succinct in your answers, especially with respect to those questions that ask for discussion. Write legibly.

Area I: Contract Theory Question (Econ 206)

Instructions: Answer all parts.

Consider the following principal-agent relation. At all points in time the principal has all the bargaining power. Among other implications of this, the principal makes the agent a TIOLI offer of employment. If the agent rejects the offer, the game ends and the payoff to each party is zero. If the agent accepts, he chooses an action $p \in [0, 1]$. Then the principal receives a stochastic benefit b. Assume b is drawn from the binary set $\{0, G\} \subset \mathbb{R}_+$, where $0 < G < \infty$. The probability that $b = G$ is p. The principal's payoff is $b - w$, where w is compensation paid the agent. The agent's payoff is $w - c(p)$, where $c : [0, 1] \to \mathbb{R}_+$ is twice continuously differentiable. Let $c'(\cdot)$ denote the derivative of $c(\cdot)$. Assume that $c'(0) = 0$ and that $c'(\cdot)$ is a strictly increasing function, with $c'(p) \to \infty$ as $p \to 1$.

(a) Characterize the first-best level of the action. Call this level p^*.

Assume, henceforth, that the agent is protected by limited liability; that is, his compensation (payment) at the end of the game must be non-negative.

(b) Assuming b is verifiable and p a hidden action, what contract would the principal offer in equilibrium?

(c) Will that contract achieve the first best? Why or why not?

Suppose, now and until further notice, that p is immediately observable to the principal, but it is not verifiable.

(d) Suppose, for this part only, that there is a delay between when the agent chooses his action and when b is realized. What is the equilibrium of this game?

Suppose, now and henceforth, that the game set forth above is infinitely repeated. Let $\delta \in (0, 1)$ be the relevant discount factor.

(e) Suppose, for this part only, that b is *not* verifiable (indeed, b may not even be observable to the agent). Derive a subgame-perfect equilibrium of the repeated game, with appropriate conditions on δ, such that the agent chooses p^* in every period on the equilibrium path.

(f) Return to the assumption that b is verifiable. How does this change your analysis in part (e)? Discuss briefly.

Suppose, now, that p is once again a hidden action. Assume that b is observable, but not verifiable.

(g) Derive a subgame-perfect equilibrium of the repeated game, with appropriate conditions on δ, in which the agent chooses $p > 0$ in every period on the equilibrium path.

Area II: Game Theory A (Econ 207B)

Instructions: Answer both of the following questions.

Question 1: Social Learning with Three Agents

- Three agents $i = A, B, C$ are bound together by a social network and can only observe the agents to whom they are connected through the network. The social network is represented by a *directed graph* in which nodes correspond to agents and agent i can observe agent j if there is an edge leading from node i to node j.
- Time is represented by a countable set of dates indexed by t. Each agent i receives a private (unobserved by others) signal ω_i before date 1. The ω_i's 's are uniformly distributed on the interval $[-1, 1]$. There are two actions $a = 0, 1$ and the homogenous payoff function is assumed to satisfy

$$u(a, \omega) = \begin{cases} 0 & \text{if} \quad a = 0 \\ \omega_A + \omega_B + \omega_C & \text{if} \quad a = 1 \end{cases}.$$

- At the beginning of each date t, agents choose actions simultaneously. Then each agent i observes the actions a_{jt} chosen by the agents to whom she is connected by the network. Agent i chooses the action a_{it} to maximize the expectation of his short-run payoff conditional on the information available.

(a) Consider the complete network, in which each agent can observe the other two. Show that the learning process comes to a halt by the end of the period 2 at the latest. Does the so-called herd always adopt the action that is optimal relative to the total information available to agents?

(b) Consider the circle, in which each agent observes one other agent. Show that alternating actions may continue beyond period 2. Will all agents choose the same action in finite time with probability one? If everyone eventually adopts the same action, can the action chosen be sub-optimal?

Question 2: A Simple Homophily Model Consider an economy consisting of two groups where n_k is the size of group $k = 1, 2$ and assume that $n_1 > n_2$. The social network is represented by an *undirected graph* (the relations between any pairs of nodes is symmetric so that each edge points in both directions). Let d_i be the number of neighbors of agent i and let $s_i \leq d_i$ be the the number of neighbors that agent i has in his own group. Then, a simply homophily index for group k is given by

$$h_k = \frac{\sum_{i \in k} s_i}{\sum_{i \in k} d_i}.$$

(a) Show that if $0 < h_k < 1$ for $k = 1, 2$ and the *average* degree in group 1 is at least as high as that of group 2 then $h_1 > h_2$.

(b) What are h_1 and h_2 when neighborhoods are formed in percentages that correspond to the shares of the two groups in the total population?

(c) How your answers to questions 1 and 2 change if the social network is represented by a *directed graph*?

Area III: Game Theory B (Econ 209A)

Instructions: Answer all three of the following questions.

Question 1: Consider the variant of the Hawk-Dove game

	D	H
D	$1, 1$	$0, 2$
H	$2, 0$	$1-c, 1-c$

(when $c > 1$ the game has the standard Hawk-Dove structure).

(a) Find the set of *all* Nash and trembling hand perfect equilibria for all values of c. Are the equilibrium strategies evolutionary stable?

(b) Let α^* be an evolutionary stable strategy. Does α^* necessarily weakly dominates every other strategy? Is it possible that some strategy weakly dominates α^*? Does it matter if α^* is pure or mixed?

Question 2: Consider a two-player extensive game of perfect information where player 1 moves only at the start of the game and player 2 moves once after player 1. Denote by S_i the set of pure strategies available for player $i = 1, 2$.

(a) Suppose that player 2 is never indifferent between pairs of outcomes, and that if the two players move simultaneously, then there exist a unique pure strategy Nash equilibrium in which the payoffs are (ω_1, ω_2). Show that player 1's payoff in any subgame perfect equilibrium of the extensive game is at least ω_1.

(b) Show by counter-examples that (i) if player 2 is indifferent between a pair of outcomes or (ii) the strategic game has a unique mixed strategy Nash equilibrium, then player 1's subgame perfect equilibrium payoff can be lower than ω_1.

Question 3: A bargaining problem is a pair $\langle S, d\rangle$ where $S \subset \mathbb{R}^2$ is compact and convex, $d \in S$ and there exists $s \in S$ such that $s_i > d_i$ for $i = 1, 2$. The set of all bargaining problems $\langle S, d\rangle$ is denoted by $\mathbf{B}$. A bargaining solution is a function $f : \mathbf{B} \to \mathbb{R}^2$ such that f assigns to each bargaining problem $\langle S, d\rangle \in \mathbf{B}$ a unique element in S.

(a) Show that Kalai bargaining solution

$$f^K(S, d) = \{s_1 = s_2 : s \in S\} \cap WPO(S)$$

does not satisfy INV.

(b) Show that Kalai-Smorodinsky bargaining solution

$$f^{KS}(S,d) = \{\frac{s_1}{\bar{s}_1} = \frac{s_2}{\bar{s}_2} : s \in S\} \cap WPO(S)$$

where $\bar{s}_i$ is player i's the maximum utility does not satisfy IIA.

Area IV: Psychology and Economics Question (Econ 219A)

Instructions: Depending on how you count, there are 36 parts to this question, so manage your time properly to have plenty of time for other material, and don't sweat not getting through this. Certainly do Part I before Part II.

Part I

Max will live for 3 periods. He has $\$y \geq 0$ to spend on consumption in his life; he has the money initially, and can freely save it at no interest. Max has no long-term discounting, so $\delta = 1$ everywhere below.

His utility functions in each of the three periods are given by:

$$u_1 = \ln(c_1 - c_0) = \ln(c_1),$$
$$u_2 = \ln(c_2 - c_1),$$
$$u_3 = \ln(c_3 + c_2),$$

where $c_0 = 0$ by assumption throughout. Max has habit-forming preferences, where his pleasure from consumption is solely relative to the previous period's consumption, and it is as if he were born with no past consumption.

Max chooses consumption levels each period, without commitment. Because he will simply choose $c_3 = y - c_1 - c_2$ in period 3, he is essentially only making choices in periods 1 and 2.

This problem simply asks you to solve for Max's behavior for each of 6 different personalities he can have, depending on his present bias, sophistication about that present bias, and projection bias. In each case, you'll be asked to solve for his lifetime consumption profile as a function of y. You should circle your answer to each part to the prediction by filling in "$(c_1^*, c_2^*, c_3^*) =$ ________". But for each case, you also must state and circle the maximization problem the person is doing in each of the two substantive periods: "In Period 2, Max will choose $c_2^* = \arg\max$ ____________", where this will of course take y and c_1 as given (since Max will have $y - c_1$ in savings). And then describe his Period-1 maximization problem similarly, where Max will clearly embed a prediction of his c_2 and c_3. That is, write "In period 1, Max will choose $c_1^* = \arg\max$ ____________" where your expression must include nothing but y.

Two important notes:

1) There should not be any quadratic equations or nasty math involved, every case ought eventually simplify to easy algebra (and the answers should always be rational numbers). If you are not able to simplify it thusly, either you are in error, or the question is in error; you should move without too much time; but

2) some maximization problems may make relatively little sense—there may not be a well-defined answer to some parts of some questions. If you decide that some part is indeterminant, say so; and

answer the questions you can. All math here is reflecting an underlying choice problem, so don't punt on giving answers to parts that there is a logical and determinant answer for.

Remember, for each of the 6 parts below, be sure to circle exactly and only each of three answers (in any order): 1) "$(c_1^*, c_2^*, c_3^*) =$ ________", 2) "In period 2, Max will choose $c_2^* = \arg\max$ ____________", 3) "In period 1, Max will choose $c_1^* = \arg\max$ ____________".

The 6 different personalities Max can have are:

a) Max is fully rational, with no present bias and no projection bias.

b) Max has no present bias, but he has full projection bias, where each period t he projects his current state of habit level, c_{t-1}, on to future periods.

c) Max has no projection bias, but is present biased, with $\beta = \frac{1}{2}$. Max is naive about this present bias.

d) Max has no projection bias, but is present biased, with $\beta = \frac{1}{2}$. Max is sophisticated about this present bias.

e) Max has both full projection bias and is present biased, with $\beta = \frac{1}{2}$. Max is naive about this present bias.

f) Max has both full projection bias and is present biased, with $\beta = \frac{1}{2}$. Max is sophisticated about this present bias.

Please don't mess up the order (e.g., reverse sophisticate vs. naivety answers); or, as a safeguard, repeat Max's personality when you present the answers. You don't need to explain or interpret your answers except if you are worried they are ambiguous—devote all your intuition to making sure you get the right answers. And always circle exactly and only intended answers.

Part II

Now we consider the same 6 personalities of Max, with the same utility function as above. But now we assume that Max lives in a world where there is 100% interest between periods. This means that if he consumes c_1 in period 1, he'll have savings of $(2(y - c_1)$ going into period 2, and (more importantly) if he consumes c_1 and c_2, then his savings entering period 3, and hence his consumption c_3, will be $s_3 = c_3 = 4y - 4c_1 - 2c_2$.

So re-do (a) through (f) in this new situation.

Economics 234A Financial economics

Consider an endowment economy where exogenous log consumption $c_t = \log C_t$ follows

$$c_{t+1} = c_t + \mu + \varepsilon_{t+1}$$

and $\varepsilon_{t+1} \sim N\left(0, \sigma^2\right)$ is i.i.d. over time. The representative consumer solves

$$\max \mathrm{E}_t \sum_{j=0}^{\infty} \delta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma}.$$

(a) Using the formula for the expected value of a lognormal random variable, compute $m = \log\left[\mathrm{E}_t C_{t+1}/C_t\right]$ the log expected growth rate of consumption.

(b) Write down a stochastic discount factor at period t. Suppose there is a riskfree asset in zero net supply and compute the log riskfree return r_f.

In this economy, the stock market can be represented as a claim to all future endowments, i.e., the Lucas tree. Denote the price of this asset in period t by P_t. For the remainder of the problem, assume that $C_t/\left(P_t + C_t\right) = b$ is a constant over time, where b can be interpreted as the consumption to wealth ratio. This assumption implies that the price of the Lucas tree is a constant multiple of the single state variable C_t.

(c) Consider the investment of purchasing the Lucas tree at time t. Express the payoff of this investment at period $t+1$ as a function of C_{t+1} and b. Express the price of this investment P_t as a function of C_t and b. Combine these to get the return on the Lucas tree $1 + R_{t+1}$. What is σ_r^2, the variance of the log return r_{t+1}?

(d) Write down the key asset pricing equation for $1 + R_{t+1}$, take logs and evaluate the expectation to compute $\log\left(1-b\right)$ with exogenous variables. Note, you can now express P_t with exogenous variables.

(e) Use the loglinear Euler equation for R_{t+1} to compute the equity premium $\mathrm{E}r_{t+1} - r_f + \sigma_r^2/2$, and combine this with (b) to get the expected stock return $\mathrm{E}R_{t+1} \approx \mathrm{E}r_{t+1} + \sigma_r^2/2$.

Now we add unlikely disasters to this model. For the rest of the problem, suppose that log consumption follows

$$c_{t+1} = c_t + \mu + \varepsilon_{t+1} + v_{t+1}$$

with $\varepsilon_{t+1} \sim N(0, \sigma^2)$ i.i.d., where v_{t+1} equals zero with probability $1-p$ and $\log\left[1-d\right]$ with probability p, $1 > d \geq 0$ and $p \geq 0$ is a small number. Here ν_{t+1} can be interpreted as an unlikely "disaster" event where consumption drops by d percent. We assume that ε and ν are independent, and that the representative consumer has CRRA preferences as above.

(f) Compute an expression for $m = \log\left[\mathrm{E}_t C_{t+1}/C_t\right]$ the log growth rate of consumption, using the fact that for small x, $\log\left(1-x\right) \approx -x$. Compare your result with (a).

(g) Write down the SDF in period t. Express r_f using the approximation for the log function the same way as in (f). Is the riskfree rate higher or lower than in (b)? Why?

(h) Consider the risky asset which is a claim to next period's total endowment C_{t+1}. Express the price of this asset, P_t, with the key asset pricing equation. Compute $\log[P_t/C_t]$.

(i) Show that $\log \mathrm{E}[1 + R_{t+1}] = \log \mathrm{E}[C_{t+1}/C_t] - \log[P_t/C_t]$, and express this quantity using your results. Compute the expected excess return $\mathrm{E}R_{t+1} - R_f \approx \log \mathrm{E}[1 + R_{t+1}] - r_f$. Compare your result to the expected excess return in (e). Is the equity premium higher now? Why?