Labor Field Exam

August 2021

There are three parts to this exam. Each should take about one hour. Calculators are not necessary. Please explain your notation in order to maximize chances of partial credit.

Write your answer to each part in a separate book.

Part 1 (244)

You are analyzing data from the national Job Corps experiment: a multi-site RCT designed to evaluate the effects of job training on employment and earnings. In this experiment access to training was lotteried at random to program applicants, each of which applied to a particular program site.

You are given site level collapses of the data. Specifically, at each site $s \in \{1, 2, ..., S\}$ you have access to the variables $(n_{s1}, n_{s0}, Y_{s1}, Y_{s0})$, where:

- n_{s1} is the number of applicants to site s lotteried into training,
- n_{s0} is the number of applicants to site s lotteried out of training,
- Y_{s1} is the number of applicants to site s lotteried into training who were employed 12 months after baseline,
- Y_{s0} is the number of applicants to site s lotteried out of training who were employed 12 months after baseline.

Modeling the applicant employment outcomes as independent binomial trials with site specific success probabilities, we assume:

$$(Y_{s1}, Y_{s0}) \,|n_{s1}, n_{s0} \sim Bin\left(n_{s1}, p_{s1}\right) \times Bin\left(n_{s0}, p_{s0}\right), \quad \text{for } s = 1, \ldots, S,$$

where p_{s1} is the potential employment probability of applicants to site s if lotteried into training and p_{s0} is the potential employment probability of applicants to site s if lotteried out of training.

Answer as many of the questions below as possible. If you get stuck, skip to the next question.

- a) Let $\beta_s = p_{s1} - p_{s0}$ denote the expected effect of being lotteried into training on the probability of employment among applicants to site s. Propose an unbiased estimator $\hat{\beta}_s$ of β_s.
- b) Derive the sampling variance V_s of $\hat{\beta}_s$. Propose an unbiased estimator $\hat{V}_s$ of V_s.
- c) Prove that $\mathbb{E}\left[\hat{\beta}_s^{\,2}\right] = \beta_s^2 + V_s$.

- d) Propose a bias-corrected estimator $\widehat{\sigma_\beta^2}$ of the cross-site variance of lottery effects $\sigma_\beta^2 = \frac{1}{S}\sum_{s=1}^{S}\beta_s^2 - \left(\frac{1}{S}\sum_{s=1}^{S}\beta_s\right)^2$. Hint: use your (squared) standard errors $\left\{\hat{V}_s\right\}_{s=1}^{S}$.

- e) Suppose that only a share $\pi < 1$ of the sites have lottery effects different from zero. Letting $D_s = 1\{\beta_s \neq 0\}$ be an indicator for site s exhibiting a non-zero effect and using $\mathbb{E}_S$ to denote the (equally weighted) average across sites, we can write $\pi = \mathbb{E}_S[D_s] = \frac{1}{S}\sum_{s=1}^{S} D_s$. Use the law of total variance to derive an expression relating σ_β^2 to π, the variance of non-zero effects $\sigma_1^2 \equiv \mathbb{V}_S(\beta_s|D_s = 1) = \frac{\sum_{s=1}^{S} D_s\beta_s^2}{S\pi} - \left(\frac{\sum_{s=1}^{S} D_s\beta_s}{S\pi}\right)^2$, and the mean of non-zero effects $\mu_1 \equiv \mathbb{E}_S[\beta_s|D_s = 1] = \frac{1}{S\pi}\sum_{s=1}^{S} D_s\beta_s$.

- f) Use your answer to the above question to derive a lower bound on π in terms of σ_β^2 and the grand mean effect $\mu = \mathbb{E}_S[\beta_s] = \pi\mu_1$.

- g) What value of μ_1 does the lower bound on π imply? Is this an upper or a lower bound on μ_1?

Part 2 (250A)

Consider the following two-period model. Workers are of two skill types, θ_L and $\theta_H > \theta_L$. A fraction λ of workers are of type θ_H. Initially, a worker observes his/her own skill level but firms do not. Before period 1, workers make a publicly-observed binary education decision $e \in \{0, 1\}$. Education has no monetary cost. Workers who choose $e = 1$ spend the first period in school and earn nothing, while workers who choose $e = 0$ start working immediately in period 1. Everyone works in period 2. In both periods, productivity for a worker of skill type θ who has education level e is

$$y(e, \theta) = e + \theta.$$

Production by workers who work in the first period is publicly observed. In each period of work a large population of firms makes wage offers to workers, competing a la Bertrand, and workers accept the highest wage offer. Workers cannot commit to work for a firm for more than one period. A firm that hires a worker of type θ and education e at wage w makes profit $\pi = y(e, \theta) - w$. There is no discounting between periods.

1. Let w_1 denote the equilibrium first-period wage offered to workers who do not get education. Let $w_2(0, \theta)$ denote the equilibrium second-period wage for workers of type θ that choose no education in the first period, and let $w_2(1)$ denote the equilibrium second-period wage for workers who chose to get education in the first period. Explain why $w_2(0, \theta)$ depends on θ, but w_1 and $w_2(1)$ do not.

2. Write out the total payoff for a worker of skill type θ that chooses education level e, as a function of the firm wage offers from part (1). Use this expression to derive the optimal education choice for a worker of type θ, $e^*(\theta)$, as a function of the wage offers.

3. Informally describe the conditions on education choices, firm wage offers, and firm beliefs required for a Perfect Bayesian Equilibrium (PBE) of this game.

4. Find conditions under which there exists a pooling PBE where no workers get education in the first period.

5. Find conditions under which there exists a separating PBE where only type θ_L gets education in the first period.

6. Show that there cannot exist a PBE in which only type θ_H gets education in the first period. Provide economic intuition for this result.

7. Compare your results from parts (6) and (7) to the predictions of classic models of signaling in education (e.g., Spence 1973). Briefly discuss the plausibility of each model, citing empirical evidence where possible. Informally discuss possible extensions that would allow for the possibility that higher-skilled workers get more education.

Part 3 (250B)

There is a pool of workers of ability A where A is distributed according to cdf $F(A)$ with positive density $f(A)$. A worker of ability A produces

$$y = A \times e$$

in output where e is the effort the worker chooses to exert. The price of output is 1. Effort is unobservable.

Workers obtain utility from earnings w and disutility from effort:

$$u(w, e) = w - c(e)$$

The worker's cost of effort is given by $c(e)$ where $c'(e) > 0$ and $c''(e) > 0$.

1. What is the efficient (first-best) level of effort and how does this relate to the ability of the worker? What is the total level of output produced when all workers exert the first-best level of effort?

2. **Hourly Wages:** Suppose workers are paid an hourly wage H, subject to a minimum output requirement y_0. Workers that produce less than y_0 are fired and not paid.

   (a) Given this compensation scheme, there is a cutoff level of ability A_0 such that all workers with ability above A_0 accept the job and all workers with ability below A_0 turn down the job. Derive an expression for A_0, the minimum level of ability for which a worker is indifferent between accepting your job and not working. Use b to denote the utility associated with leisure.

   (b) Compare the effort choices of (1) the lowest ability worker (i.e. a worker with ability A_0) and (2) the highest ability worker to the efficient levels you found in part 1.

   (c) Explain what it means to say that workers with ability levels above A_0 earn rents.

   (d) Suppose workers have the option of working for an outside firm, which provides them with $O(A)$ in utility. $O'(A) > 0$ so higher ability workers have better outside options. Define A_h to be the level of ability at which a worker is indifferent between working at this firm (under the hourly wage scheme) and taking her outside option. Write the indifference condition.

3. **Piece Rates:** Consider now a piece-rate scheme, in which workers receive $by - K$ per unit of output y.

   (a) What is the effort choice for a worker of ability A under this scheme?

   (b) Explain, intuitively, why you might prefer (or not prefer) a piece rate scheme in this setting.

4. **Piece Rate with a Guarantee:** Suppose that, instead of offering a pure piece rate, you offer workers a combination of a piece rate, with a minimum guarantee. Specifically you offer workers

   $$\max\{by - K, H\}$$

   with the same minimum standard as before (workers producing less than y_0 earn nothing). For simplicity, assume workers only have the choice of working at this firm or engaging in leisure.

   (a) How does output change when you switch to this scheme (compared to the hourly rate in part 2)?

   (b) What happens to the average of workers' ability? What about the variance of output/worker?

5. Relate your findings in part 4 to the empirical work on performance pay that we discussed in class.