

Labor Economics

August 2023

Please answer all three questions on this exam. You should plan to spend about one hour per question. Please write your answers for each question in a separate book.

Econ 250A: Monopsony and efficiency wages

Consider the wage posting decision of a firm that provides daycare services. To provide these services, the firm employs “nannies.” Suppose there is a continuum of potential nannies of measure one. Nannies differ only in their outside options, which are distributed according to the twice differentiable CDF F . Consequently, $F(w)$ nannies will be willing to work for the firm at wage w .

The firm’s profits can be written

$$\pi(w) = F(w) [p(w) - w],$$

where $p(w)$ gives the average productivity of an employed nanny, which can potentially be influenced by the wage.

a) Derive the optimal wage in terms of average productivity $p(w)$, the productivity-wage elasticity $\rho = d \ln p / d \ln w$, and the labor supply elasticity $\eta = d \ln F / d \ln w$.

b) Suppose $p(w) = R(F(w)) / F(w)$, where $R(F(w))$ is the revenue that results from employing $F(w)$ nannies. Define the revenue-employment elasticity $\varepsilon = d \ln R / d \ln F > 0$. Write ρ in terms of ε and η . Why might you expect $\varepsilon < 1$?

c) Use your answer above to derive an expression for the firm’s profit margin $\frac{p(w)-w}{p(w)}$. How do profit margins vary with ε ? Provide intuition for your answer.

d) Use your answer to b) along with the definition of ε to write the optimal nanny wage in terms of η and the marginal revenue product of labor $R'(F(w))$.

e) Derive the limiting optimal nanny wage as $\eta \rightarrow \infty$. Provide intuition for this result.

f) Efficiency wage models stipulate that raising wages can directly boost worker productivity. To capture this idea, we can write revenue as $R(F(w), w)$. Define the elasticities $\varepsilon = \partial \ln R / \partial \ln F > 0$ and $\omega = \partial \ln R / \partial \ln w > 0$. Write ρ in terms of ε, η , and ω .

g) Write the profit margin $\frac{p(w)-w}{p(w)}$ in terms of the elasticities $(\varepsilon, \omega, \eta)$. Do efficiency wages augment or mute firm profitability? Why?

h) Express the optimal nanny wage in terms of $p(w)$ and the elasticities $(\varepsilon, \omega, \eta)$.

Econ 250B: Wage Rigidity and Morale

In class we discussed the literature on wage rigidity.

One of the most influential works on this topic is a book by Truman Bewley, based on interviews with employers and employees. Truman Bewley finds that one reason for downward nominal wage rigidity is “morale”. This also leads firms not to want to hire “overqualified” workers. This problem has you work through a model of morale.

Please assume that time is discrete, all agents are risk neutral with discount factor β . Workers’ outside options are 0. A firm has two workers. The two workers produce output y each period. However, in every

period there is a probability q that one of the workers needs to exert an additional effort at cost e in order to produce the good. If effort is exerted, output is y ; if it is not exerted when it is required, no output is produced. The firm does not know whether effort is required or whether it is exerted. The firm faces a constant probability of dying p . Workers cannot be paid a negative wage, but each worker's wage can be conditioned on the history of the output of the firm.

1. Characterize a low morale equilibrium in which workers receive a wage w when output is equal to y and in which they expect the other worker not to exert effort when there is need. Find the maximum profits for the firm in this case.
2. Characterize a high morale equilibrium in which workers expect the other worker to exert effort when needed as long as the history of output does not contain any 0 [thus workers are using trigger strategies]. What is the condition for the firm to prefer the high morale equilibrium?
3. Suppose the firm receives a negative shock such that p increases to $p' > p$. Show that the firm may actually have to pay higher wages in this case. Carefully explain the intuition. What does this imply about the behavior of wages over the cycle.?
4. Suppose that the firm needs to hire a worker (say one of the workers quit unexpectedly). There are two candidates; A and B. A has the right qualifications and thus will produce y and will never quit. B is over-qualified. With B in place the firm will produce $y'' > y$ but B also has a probability s of quitting and taking a better job in every period. Show that the firm may prefer to hire A.
5. Summarize the literature on downward nominal wage rigidities, as discussed in class.

Econ 244: OLS and IV

You are interested in the Ordinary Least Squares (OLS) regression

$$Y_i = X_i' \beta + \gamma W_i + \epsilon_i,$$

where Y_i is an outcome for individual i , X_i is a $K \times 1$ vector of regressors (including a constant) with coefficient vector β , W_i is a scalar control variable with coefficient γ , and ϵ_i is a regression residual that satisfies $E[X_i \epsilon_i] = E[W_i \epsilon_i] = 0$ by definition. You are interested in estimating β .

A. Suppose the control variable W_i is not observed. Under what conditions will the population regression of Y_i and X_i recover β ?

B. You have access to a random sample of size N on (Y_i, X_i) . Suppose you are worried that the conditions from part A are violated, so you do not trust the regression of Y_i on X_i . Your sample includes data on an additional $M \times 1$ vector Z_i , where $M \geq K$ (assume Z_i includes a constant). Let Z denote the $N \times M$ matrix collecting observations on Z , and define an *instrument matrix* $\tilde{Z} = ZA$, where A is an $M \times K$ matrix with full column rank. Define the instrumental variables (IV) estimator:

$$\hat{\beta}_{IV} = \left(\tilde{Z}' X \right)^{-1} \tilde{Z}' Y,$$

where Y is the $N \times 1$ vector collecting observations on Y_i and X is the $N \times K$ matrix collecting observations on X_i . Provide conditions under which $\hat{\beta}_{IV}$ is a consistent estimator of β . Prove that $\hat{\beta}_{IV}$ is consistent under these conditions. Assume these conditions hold for the remainder of the question.

C. You also consider a two-stage least squares (2SLS) approach in which you first fit the system of OLS regressions:

$$X = Z\Pi + \eta,$$

form fitted values $Z\hat{\Pi}$, then regress Y on $Z\hat{\Pi}$. Show that 2SLS is an IV estimator as defined in part (B). What are the instruments \tilde{Z} for 2SLS?

D. Suppose the composite residual $e_i = \gamma W_i + \epsilon_i$ is homoskedastic: $Var(e_i|Z_i) = \sigma^2$. Derive the asymptotic variance of the 2SLS estimator.

E. Now suppose your sample does not include data on X_i . However, you have access to a second random sample that contains data on X_i and Z_i , but not Y_i . You use the second sample to compute a first-stage estimate $\hat{\Pi}_2$, and form fitted values $Z\hat{\Pi}_2$ in the first sample. Consider two strategies for using these cross-sample fitted values to estimate β :

(i) An IV estimator using $Z\hat{\Pi}_2$ as the instrument matrix.

(ii) An OLS regression of Y on $Z\hat{\Pi}_2$.

Which of these two strategies produces a consistent estimate of β ? Show derivations to justify your answer.

F. Which of the two estimators from part E has lower asymptotic variance? Provide some intuition for your answer, even if you cannot formally prove it.