

Labor Economics

There are three parts of the exam. Please answer all three parts. You should plan to spend about 1 hour per question. *Use equations and graphs whenever possible, but be sure to explain your notation.*

PLEASE WRITE YOUR ANSWERS FOR EACH PART IN A SEPARATE BOOK

Part I. Answer all parts of this question

1. Consider an individual who faces a sequence of real wages in the current and future periods, w_t, w_{t+1}, \dots, w_T , and has current (real) assets A_t . (For simplicity assume that the only source of income is labor income.) Assume the individual has a per-period utility function $u(c_t, h_t; a_t)$ where $\{a_t\}$ is a sequence of preference shifters over the lifecycle. Assume in addition that the individual discounts the future at a constant rate $\beta < 1$, and can earn a real interest rate r_t on savings from period t to $t+1$.

a) Write down the Bellman equation defining the value of an optimal lifecycle plan starting in period t , $V_t(A_t)$.

b) Define $\lambda_t = V'_t(A_t)$. What is the relationship between λ_t and $E_t[\lambda_{t+1}]$?

c) Explain what is meant by "the intertemporal elasticity of labor supply" in the context of this model.

c) Suppose that

$$u(c_t, h_t; a_t) = \varphi(c_t) - \frac{a_t \eta}{1 + \eta} h_t^{\frac{1+\eta}{\eta}},$$

i.e. within-period utility is separable in consumption and leisure, and preference shocks do not affect the marginal utility of consumption. Show that the intertemporal elasticity of labor supply in this case is constant. What is its value?

d) Using the assumptions of part (c), show that the change in the optimal choice of hours from period to $t-1$ to t can be written as:

$$\Delta \log h_t = \alpha + \eta \Delta \log w_t + \varepsilon_t.$$

What terms are included in ε_t ? Discuss the likely biases in an OLS approach for estimating η .

e) Suppose that professors have non-stochastic and constant real wages. Assuming that preferences are as described in part (c), what would you have to assume about the preference shocks to explain the phenomenon of "retirement", where people stop working and don't return to work later?

2. Suppose that economy-wide real output (y_t) depends on inputs of capital (K_t) and various types of labor $L_{1t}, L_{2t}, \dots, L_{Jt}$:

$$y_t = AK^{1-\alpha} L^\alpha, \text{ where } L = h(L_1, L_2, \dots, L_K).$$

a) Show that if the price of capital in period t , r_t , is constant then output is linear in the "labor aggregate" L , and does not depend on the inputs of any particular subgroup of labor.

b) Suppose that there are only two types of labor, skilled and unskilled, and that $h(\cdot)$ is CES:

$$h(L_{1t}, L_{2t}) = [\theta_{1t} L_{1t}^{-\rho} + \theta_{2t} L_{2t}^{-\rho}]^{1/\rho}$$

where $\rho = (\sigma - 1)/\sigma$ and σ is the elasticity of substitution between skill groups. Explain what is meant by "**skill-biased technical change**" in the context of this model.

c) Assuming the model of part (b), explain how one could estimate ρ using data on relative wages and relative employment of groups 1 and 2. Carefully explain what you are assuming about θ_{1t} and θ_{2t} .

d) Suppose that there are 2 types of skilled workers, male ("M") and female ("F"), and also two types of unskilled workers, male and female. Assume that h is a "nested CES"

$$\begin{aligned} h(L_{1t}, L_{2t}) &= [\theta_{1t} L_{1t}^{-\rho} + \theta_{2t} L_{2t}^{-\rho}]^{1/\rho} \\ L_{1t} &= [a_{Ft} L_{1Ft}^{-\tau} + a_{Mt} L_{1Mt}^{-\tau}]^{1/\tau}, \\ L_{2t} &= [b_{Ft} L_{2Ft}^{-\tau} + b_{Mt} L_{2Mt}^{-\tau}]^{1/\tau} \end{aligned}$$

Show how you can estimate the parameters ρ and τ using data on wages and employment of the various skill groups. What do you have to assume about the terms a_{Ft} , a_{Mt} , b_{Ft} , and b_{Mt} in your proposed strategy?

e) An economist has suggested that over the past 3 decades, technical changes have made women more productive than men. What would you expect to see in the data if this were true?

Part II – Oaxaca - Blinder Redux

Consider an economy where each individual's wages are determined by one of two skill pricing regimes: “male” or “female”. These regimes can be represented by the system:

$$\begin{aligned} Y_i^m &= \alpha^m + S_i \beta^m + p^m \varepsilon_i \\ Y_i^f &= \alpha^f + S_i \beta^f + p^f \varepsilon_i \end{aligned}$$

where Y_i^m gives the log wage of individual i under the male pricing regime, Y_i^f gives i 's log wage under the female pricing regime, S_i denotes i 's years of schooling, and ε_i denotes individual i 's unobserved skill level. The parameters (α^m, α^f) are scalar intercepts, (β^m, β^f) are scalar “returns” to schooling, and (p^m, p^f) are the scalar “prices” of skill under the gender specific regimes. Suppose additionally that:

$$E[\varepsilon_i | S_i] = 0.$$

Assignment to these regimes involves some randomness. Let the random variable G_i evaluate to m in the event that the individual is a man and f in the event that the individual is a woman. We can now define observed wages as:

$$Y_i \equiv 1[G_i = m] Y_i^m + 1[G_i = f] Y_i^f.$$

Assume that we have access to a random sample of pairs $\{Y_i, G_i\}_{i=1}^N$.

Questions

- a) Comment on the interpretation of the following conditional independence assumption (CIA):

$$\varepsilon_i \perp G_i | S_i.$$

What are some reasons why this condition might fail?

- b) Use the CIA assumption to prove that $(\alpha^m, \alpha^f, \beta^m, \beta^f)$ are identified along with the ratio p^m/p^f .
 c) Provide an interpretation of α^m and α^f . Does the interpretation change if we measure schooling in different units? (e.g. years since 6th grade?)
 d) Suppose that, for $g \in \{m, f\}$, the population values of (α^g, β^g) and $\mu^g \equiv E[S_i | G_i = g]$ are given by:

	Mean Schooling (μ^g)	Return (β^g)	Intercept (α^g)
Male	10	.12	2
Female	11	.10	1.9

Derive the impact on women's mean wages of replacing the female distribution of schooling $F_{S|G=f}(\cdot)$ with the male distribution $F_{S|G=m}(\cdot)$.

- e) Derive the impact on women's mean wages of changing the female return to schooling from β^f to β^m .
 f) Derive the impact on women's mean wages of giving them both the male return to schooling and the male schooling distribution $F_{S|G=m}(\cdot)$.
 g) How does this compare to the sum of your answer answers from parts e) and f)? Why?

Part III – Job Market Signaling

Consider the following two-period model. Workers have heterogenous unobserved ability, denoted θ . Ability is uniformly distributed on the interval $[0, 1]$. In the first period of the model, workers choose a schooling level $e \in \{0, 1\}$. The cost of schooling level e for a worker with ability θ is ce/θ , with $c > 0$. In the second period, a large number of firms observe schooling *but not ability*, and compete a la Bertrand to hire workers. Productivity in period 2 for a worker with ability θ and schooling e is given by

$$y(\theta, e) = \theta(1 + e).$$

There is no discounting between periods.

1. Define a Perfect Bayesian Equilibrium (PBE) of this game.
2. Characterize a pooling PBE in which no workers obtain schooling. Under what conditions does such an equilibrium exist?
3. Now consider a separating equilibrium where some workers obtain schooling and others don't. Show that any separating equilibrium must involve a cutoff ability level $\bar{\theta}$, such that workers with ability above $\bar{\theta}$ obtain schooling and workers below $\bar{\theta}$ don't.
4. Characterize firms' wage offers as a function of the cutoff $\bar{\theta}$. Solve for $\bar{\theta}$ and give conditions under which a separating equilibrium exists.
5. Now suppose there is a third period in which ability is observed. Productivity in period 3 is again $y(e, \theta) = \theta(1 + e)$. Give new conditions under which a separating equilibrium exists. Are these more or less strict than the conditions in part (4)? Why?
6. Now suppose you have data on third-period wages and education for a sample of individuals in the separating equilibrium from part (5). You are interested in estimating the effect of education on third-period wages. What is the population average treatment effect of education on third-period wages in this model (ATE)? What is the effect of treatment on the treated (TOT)? How does the ordinary least squares (OLS) coefficient from a regression of wages on education compare to these causal parameters?
7. You decide to run a randomized experiment to estimate the returns to schooling. Treated individuals in this experiment are offered a full scholarship (so schooling becomes costless), while control individuals receive no aid. (Assume your sample is small enough to be negligible relative to the full population.) Describe an estimation strategy that uses data from this experiment. What parameter does your strategy recover? How does this parameter compare to the ATE, the TOT, and the OLS return to schooling in the model above? Give some intuition.
8. It turns out that instrumental variables (IV) estimates of the return to schooling often exceed OLS estimates. How is this related to your answer in part (7)? Discuss three reasons why IV estimates might exceed OLS in practice. What is the most plausible explanation, in your view?