

**International Economics**  
**Field Exam**

August 2018

**GENERAL INSTRUCTIONS:**

This is a three-hour field exam. There are five questions in total but you only need to answer three questions. Question 1 corresponds to course 280A, question 2 corresponds to course 280B, questions 3 and 4 correspond to 280C, and question 5 corresponds to 280D. You need to answer questions from three courses, and if you choose to answer questions from 280C then you need only answer one of them, so either 3 or 4. Note that you have 3 hours for this exam, so you have 1 hour average for each question.

**Question 1**

This is a question about what happens when we combine the the Hecksher-Ohlin model with gravity.

1. Why is it that we don't have a gravity equation in the Hecksher-Ohlin model?
2. Consider a multi-sector Armington model, but instead of our usual assumption that labor is the only factor of production, we now have two primary factors: labor and capital, available in fixed quantities and freely mobile across sectors but immobile across countries. Assume that production of good  $s$  in country  $i$  takes place according to  $Q_{i,s} = A_{i,s}L_{i,s}^{\alpha_s}K_{i,s}^{1-\alpha_s}$ , with  $\alpha_s \in (0, 1)$ . Let  $\sigma_s > 1$  be the elasticity of substitution across goods from different origins in sector  $s$ , and assume that upper-tier preferences are Cobb-Douglas with expenditure shares  $\gamma_{j,s}$ . Let  $\tau_{ij,s}$  be the iceberg trade costs in sector  $s$  for trade flows from  $i$  to  $j$ . Finally, let  $w_i$  and  $r_i$  be the factor prices for labor and capital in country  $i$ .
  - (a) What is the expression for trade shares  $\lambda_{ij,s} \equiv X_{ij,s}/X_{j,s}$ ?
  - (b) Assuming that  $\ln \tau_{ij,s} = \ln \eta_{ij,s} + \xi_{ij,s}$ , where  $\eta_{ij,s}$  is observable, imagine we write down gravity with fixed effects as follows:

$$\ln X_{ij,s} = \delta_{i,s}^o + \delta_{j,s}^d + \beta_s \ln \eta_{ij,s} + \varepsilon_{ij,s}.$$

What is the structural counterpart of  $\delta_{i,s}^o$ ,  $\delta_{j,s}^d$ ,  $\beta_s$  and  $\varepsilon_{ij,s}$ ? Where are the Hecksher-Ohlin forces absorbed in this gravity equation?

- (c) Assume that  $\alpha_s = \alpha$  for all  $s$ . Derive the formula for the gains from trade.
- (d) Intuitively, how do you think that variation in  $\alpha_s$  across  $s$  would affect the gains from trade relative to the formula in (2c)?

- (e) How is your answer in (2d) related to the way in which curvature (i.e., higher heterogeneity in sector-level productivities across workers) affects the gains from trade in the multi-sector EK-Roy model in “Slicing the Pie” (Galle, Rodriguez-Clare, Yi, 2017)?
3. How are the factor-price insensitivity (FPI), Rybczynski, and Stolper-Samuelson theorems affected by adding gravity to the Hecksher-Ohlin model as in (2)?

## Question 2

Consider two endowment economies, Home and Foreign. Let  $*$  denote Foreign variables. Time is continuous and there is no aggregate uncertainty. Home endowment, denoted  $X_t$ , grows exogenously at rate  $g$  (i.e.  $dX_t/dt = gX_t$ ). Foreign endowment, denoted  $X_t^*$ , grows at the same rate. Each endowment is produced by an ‘orchard’ of Lucas trees in each country, where each individual tree produces one unit of the respective ‘fruit’, so  $X_t$  denotes also the number of trees at time  $t$ . At each instant of time, a fraction  $\rho$  of existing trees ‘dies’ of obsolescence while  $(\rho + g)X_t$  new trees appear. There is a market for claims to the trees in the Lucas orchard. The owner of a tree owns a claim to  $\delta \leq 1$  of the fruit produced by the tree, as long as the tree is alive. The remaining  $1 - \delta$  fruit is distributed to newborns (see below). Therefore, the aggregate dividend income produced by the Home orchard is  $\delta X_t$ . Denote  $v_t$  the value of a claim to a Home Lucas tree, in the domestic currency, and  $V_t = X_t v_t$  the value of the entire Home ‘orchard’ (also in domestic currency). There is also a government that issues nominal public debt  $D_t = dX_t$ , proportional to output and financed by raising taxes  $\tau$  on the non-financial income of newborns. The value of all domestic assets is  $V_t + D_t$ .

In each country, people are born and die at a constant i.i.d. rate  $\theta > 0$ , so that population remains constant and normalized to 1. We assume that people only consume at the time of their death: until then, they save and re-invest all their income. Since death is random, aggregate nominal consumption expenditures is simply  $C_t = \theta W_t$  where  $W_t$  denotes Home financial wealth in Home currency. Preferences are such that people in each country spend a fraction  $1/2$  on the home good and the remaining fraction on the foreign good (there is no home-bias and countries are symmetric). With these assumptions, the income of newborn is  $(1 - \tau)(1 - \delta)X + (\rho + g)X_t v_t$ .

We assume that prices are permanently rigid in each country in the producer’s currency (producer currency pricing) and normalized to 1. In each country there is a monetary authority that follows a Taylor rule but is constrained by the Zero-Lower-Bound (ZLB): the nominal interest rate  $i_t$  is positive if output is at its potential (i.e.  $X_t$ ) but constrained at  $i_t = 0$ . At the ZLB, we will show that output falls below potential, to  $\xi_t X_t$  where  $0 < \xi_t < 1$ . What this means is that the owners of a fruit can only sell a fraction  $\xi$  of that fruit.

1. Consider first the case where each country is in financial autarky. Denote  $i^a$  the autarky nominal interest rate. Explain why the following no-arbitrage relationship must hold for the value of a single tree:

$$i^a v_t = -\rho v_t + \delta \xi_t + \dot{v}_t$$

2. Along a Balanced-Growth-Path (BGP), the economy is growing at a constant rate  $g$  and the exchange rate is constant. It follows that all prices are constant and the nominal interest rate  $i^a$  coincides with the real interest rate  $r^a$ . From the previous question, explain why the value of the Lucas Orchard satisfies:

$$r^a V_t = -\rho V_t + \delta \xi_t X_t$$

3. Explain why, along the BGP, the aggregate wealth accumulation equation satisfies:

$$gW_t = -\theta W_t + (1 - \tau)(1 - \delta)\xi_t X_t + (\rho + g)V_t + r^a W_t$$

4. Along the BGP, public debt satisfies (you are not asked to show this):

$$(r^a - g)d = (1 - \tau)(1 - \delta)\xi.$$

Under financial autarky, total home asset demand  $W_t$  must equal total home asset supply  $V_t + D_t$ . Show that, if the economy is outside the ZLB (i.e.  $\xi = 1$ ), the equilibrium real rate at Home is equal to the ‘autarky natural real rate’ defined as:

$$r^{a,n} \equiv -\rho + \frac{\delta\theta}{1 - \theta d}$$

Discuss conditions under which Home can avoid the ZLB under financial autarky in terms of the underlying parameters.

5. Show that if the Home economy is at the ZLB, output is determined by:

$$\xi^a = 1 + \frac{1 - \theta d r^{a,n}}{1 - \frac{\delta\theta}{\rho}} < 1$$

Explain why we have a recession in the Home country in terms of asset demand/asset supply and/or in terms of goods demand/goods supply. In particular, discuss the role of public debt  $D = dX$ .

6. Using the market clearing condition for Home and Foreign goods, show that the autarky nominal exchange, defined as the Home price of the Foreign currency, satisfies:

$$E^a = \frac{\xi^a}{\xi^{a*}}$$

7. Consider now the case of financial integration. Explain why, along a BGP, the exchange rate must remain constant, and nominal (and real) interest rates must be equated in both countries. We denote  $r^w$  the world real interest rate.
8. Following similar steps as 2)-4) above, show that outside the ZLB, the world real interest rate satisfies:

$$r^{w,n} \equiv -\rho + \frac{\bar{\delta}\theta}{1 - \theta \bar{d}}$$

where  $\bar{z} = 0.5(z + z^*)$ . Discuss whether a country could escape the ZLB under financial integration, but not under financial autarky, or vice versa.

9. Define net foreign assets as  $NA = W - (V + D)$ . Show that, outside the ZLB, Home's net foreign assets satisfy:

$$\frac{NFA}{X} = \frac{(1 - \theta d)(r^{w,n} - r^{a,n})}{(g + \theta - r^{w,n})(\rho + r^{w,n})}$$

Explain what drives global imbalances outside the ZLB. You may want to use a Metzler-Diagram.

10. Assume now that the world is in a global liquidity trap ( $r^w = 0$ ). Discuss informally whether there is a unique equilibrium in terms of the Home and Foreign output gaps ( $\xi, \xi^*$ ) and nominal exchange rate  $E$ . Is there a potential for currency wars? Without derivations, explain what drives global imbalances at the global ZLB.

### Question 3

Consider two countries, Home and Foreign. Let  $*$  denote foreign variables. There are two time periods,  $t = 1, 2$  and no uncertainty. Home is populated by agents with preferences given by:

$$U(c_1, c_2) = u(c_1) + \beta u(c_2).$$

Home agents receive an endowment of the single, tradable good each period. Let  $y_1$  and  $y_2$  denote the endowment in the respective periods. Let the price of the good be normalized to 1 in foreign currency. Let  $e_t$  be units of home currency per foreign currency in period  $t$ . We assume the law of one price holds, so the domestic currency price of the good is  $e_t$ . Home agents pay lump-sum taxes in domestic currency  $e_t T_t$ ,  $t = 1, 2$ . Home agents can invest in domestic government bonds and foreign bonds. Let  $a$  denote their holding of domestic assets and  $f$  denote their foreign assets, both purchased in period 1 out of the endowment  $y_1$ . Domestic assets are nominal bonds that cost one unit of *domestic* currency in period 1 and pay  $1 + i$  units of domestic currency in period 2. Foreign bonds cost a unit of *foreign* currency in period 1 and pay  $1 + i^*$  units of foreign currency in period 2. We impose the borrowing constraint  $f \geq 0$  for Home agents. There is no constraint on  $a$ . Home's government issues domestic currency bonds  $B$  that pay the equilibrium rate  $i$ . They hold foreign reserves  $F$  earning  $i^*$ . They impose lump-sum taxes on Home agents  $T_t$  in period  $t$  in units of the good. Their budget constraint is:

$$F = \frac{B}{e_1} + T_1$$

$$(1 + i) \frac{B}{e_2} = (1 + i^*)F + T_2,$$

We require  $F \geq 0$ .

Foreign agents are risk-neutral and solve a portfolio problem that maximizes wealth in period 2 (in foreign currency). Specifically, they have wealth  $w$  in foreign currency in period 1 and use this to purchase Home bonds ( $a^*$ ) and Foreign bonds ( $f^*$ ). They are subject to short-selling constraints  $a^* \geq 0$  and  $f^* \geq 0$ . We assume that the Foreign bond market clears at  $1 + i^*$  regardless of choices made by Home and Foreign agents. This can be supported by a separate group of foreign agents that do not trade internationally but

pin down the foreign currency interest rate. The Home bond market clearing condition is:

$$B = a + a^*.$$

1. Write the Home agent's problem in a competitive equilibrium given  $\{e_1, e_2, i, i^*\}$  including all constraints.
2. Use the first-order conditions to derive an Euler Equation in terms of the Home asset's interest rate.
3. What is the analogous condition for the Foreign interest rate?
4. Argue that Uncovered Interest Parity (UIP) holds with weak inequality:

$$1 + i \geq (1 + i^*) \frac{e_2}{e_1}$$

When does UIP hold with equality?

5. Write the Foreign agent's portfolio problem. Is it ever the case in equilibrium that  $f^* > 0$  and the constraint  $a^* \geq 0$  strictly binds? What about vice versa?
6. Show that in a Competitive Equilibrium we have:

$$c_1 + \frac{c_2}{1 + i^*} = y_1 + \frac{y_2}{1 + i^*} - \frac{a^*}{e_1} \left[ \left( \frac{1 + i}{1 + i^*} \right) \frac{e_1}{e_2} - 1 \right]. \quad (1)$$

Interpret this expression.

7. Suppose the Home government can choose the Home agent's allocation as well as  $\{i, e_1, e_2\}$ . However, the allocation and prices must satisfy budget sets, market clearing and the Foreign agents' Euler equations. Write down this problem and argue that the only effective constraints are

$$c_1 + \frac{c_2}{1 + i^*} \leq y_1 + \frac{y_2}{1 + i^*}$$

$$c_1 \leq y_1 + w.$$

Let  $\{\hat{c}_1, \hat{c}_2\}$  be the solution to this planning problem. Assume

$$w > \hat{c}_1 - y_1; \quad (2)$$

that is, the last constraint does not bind for the planning problem. For what follows, assume that the government does not necessarily set policy optimally.

8. Assume that the Home government is prevented from holding foreign assets (that is, we restrict  $F = 0$ ). Show that in any competitive equilibrium the Uncovered Interest Parity (UIP) condition holds with equality:

$$1 + i = (1 + i^*) \frac{e_2}{e_1}.$$

Hint: Replace the equal sign with a strict inequality and generate a contradiction. Use:

- i. Use equation (1) and the Euler Equation to argue that  $c_1 < \hat{c}_1$
  - ii. Consider whether  $c_1 < \hat{c}_1$ , bond market equilibrium and (2) imply a contradiction.
9. Relax the constraint on  $F$  so that any  $F \geq 0$  is possible. Suppose that we impose a zero lower bound restriction  $i \geq 0$  and the government fixes  $\{e_1, e_2\}$  at some values (say,  $e_1 = e_2 = \bar{e}$ ). Show that it is possible in equilibrium that UIP may fail when the zero lower bound strictly binds. In which direction?
10. What role does  $F$  play in generating the failure of UIP?

#### Question 4

Note: all variables are in logs and I use the conventional notation from my lecture notes.

1. Consider the UIP condition:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t. \quad (3)$$

- (a) Explain this equation, the related Fama regression and the UIP puzzle.
- (b) What is the UIP shock  $\psi_t$  (provide a few economic explanations)? How does it help resolve the UIP puzzle?
- (c) Explain why equation (3) is the equilibrium condition on exchange rate imposed by the financial market? The financial markets is only concerned with  $\Delta e_{t+1}$ , not with the level of  $e_t$ ; why?

2. Consider the real exchange rate (RER):

$$q_t = e_t + p_t^* - p_t. \quad (4)$$

- (a) What are the empirical properties of RER?
- (b) What is the PPP puzzle? What are the typical approaches to resolve the PPP puzzle?
- (c) What is the relationship between the RER and the terms of trade?

3. Consider the country budget constraint:

$$\beta b_{t+1} - b_t = nx_t = \lambda q_t - \xi_t, \quad (5)$$

where  $b_t$  is net foreign assets,  $\beta = 1/R$  is discount factor,  $nx_t$  is net exports and  $\xi_t$  is the summary of all shocks that shift net exports holding RER constant.

- (a) Explain this equation and the implied assumptions. Why does  $nx_t$  increase with  $q_t$  and how empirically relevant is this relationship?

- (b) Explain the macroeconomic discipline that this equation imposes on the equilibrium path of the exchange rate. Contrast your answer with that in 1(c).

4. Assume that monetary policy can achieve  $i_t = i_t^* = p_t = p_t^* = 0$ .<sup>1</sup> Further assume that  $\psi_t$  follows an AR(1) process with persistence  $\rho$  (i.e.,  $\psi_t = \rho\psi_{t-1} + \varepsilon_t$ ), while  $\xi_t \equiv 0$ .

- (a) With these assumptions, combine (3)–(5) into a two-equation dynamic system in  $(e_t, b_t)$  and solve it (using your favorite approach) to obtain:

$$\Delta e_t = \frac{\beta}{1 - \beta\rho} \left( \psi_t - \frac{1}{\beta}\psi_{t-1} \right).$$

If you get stuck with the solution, just describe the logical steps and use the above equation as the result.

- (b) Explain why the above solution for  $e_t$  is an ARIMA(1,1,1) process. Show that  $e_t$  follows approximately a random walk when  $\beta, \rho \approx 1$ .
- (c) Explain intuitively the transmission of the financial shock  $\psi_t$  into the equilibrium dynamics of the exchange rate using equilibrium conditions (3) and (5).

5. Describe the origin and implication of the theoretical Backus-Smith condition. Why is there a Backus-Smith puzzle? What are some of the possible solutions to the Backus-Smith puzzle?

### Question 5

Consider the spatial equilibrium model of Allen and Arkolakis (2014). Consumers consume a traded good, and have CES demand over varieties of this good, with elasticity of substitution between goods noted  $\sigma$ :

$$C = \left[ \sum_i c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Labor is the only factor of production. Trade between regions is costly (iceberg trade costs between  $i$  and  $j$  are noted  $\tau_{ij}$ ) and is modelled following the Armington assumption. Locations are characterized by an exogenous productivity shifter  $T_i$  and an exogenous amenity shifter  $A_i$ . Productivity is subject to agglomeration externality: we assume it responds to the population of a location with elasticity  $\alpha$ . That is, the production function in region  $i$  is:  $Y_i = T_i (L_i)^\alpha L_i$ . Amenity responds to the population of a location with elasticity  $\beta$ . That is, utility in region  $i$  is:  $u_i = A_i (L_i)^\beta$ .

1. List the endogenous variables of the model. Write down the set of equations that summarize an equilibrium of the model.

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<sup>1</sup>This is a short-cut, which gives a reasonable benchmark for thinking about exchange rates without modeling the full general equilibrium (for a complete GE model see Itskhoki and Mukhin 2017).

2. How would your answer to (a) differ if workers were immobile, and the distribution of population  $\{L_i\}$  was given and exogenous?
3. We still consider here the case where workers are immobile. Assume that there is a change in some of the fundamental parameters of the model, e.g. trade costs or productivity shifters, and that we can measure this change. Using the hat notation:  $\hat{x} = \frac{x'}{x}$  and the DEK methodology, list the equations that determine the new equilibrium expressed in changes compared to the initial equilibrium (no need to show intermediate derivations). That is, write down the set of equations that determine changes in wages, prices, trade shares and utilities  $\{\hat{w}_i, \hat{P}_i, \hat{\pi}_{in}, \hat{u}_i\}$  as a function of observable characteristics of the initial equilibrium.
4. Now, move back to the general case with labor mobility. Write down the equations that determine the new equilibrium expressed in changes compared to the initial equilibrium. That is, write down the set of equations that determine  $\{\hat{w}_i, \hat{L}_i, \hat{P}_i, \hat{\pi}_{in}, \hat{U}\}$  as a function of observable characteristics of the initial equilibrium. You can of course simply refer to the equations listed in (c) when relevant.
5. List the data needed to compute the change in welfare in this economy following a known change in trade costs.