Consider a procurement auction run by a scoring auction. A scoring auction is an auction in which the allocation of the project is determined not just by prices, but a combination of the prices (p) and quality of bidders' proposals (q). An example would be an auction for a paving job in which the bidders submit a price and the date by which the bidder promises to finish the job. A scoring rule, $S(p, q)$ is a mapping from (p, q) to a real number that determines allocation. Here, we consider a particular class of scoring rules, $S(p, q) = V(q) - p$, for some *known* function $V(\cdot)$. $V(\cdot)$ is specified by the auctioneer in advance. Allocation of the project is determined by which bidder has the highest score. The bidder with the highest score receives payment p. For this problem focus on the case when q is a scalar.

Assume that each bidder ($i = 1, ..., N$) has a cost function $C(q; \theta_i)$, and the payoff from winning the auction is $p - C(q; \theta_i)$. θ_i is the type of the bidder, drawn independently from $F_\theta(\cdot)$. The bidder knows its own type but not the type of other firms. The payoff from losing the auction is 0. The firm's problem is

$$\max_{p,q}(p - C(q; \theta_i)) \Pr\left(V(q) - p \geq \max_j(V(q_j) - p_j)\right). \tag{1}$$

Assume that $C(q; \theta)$ is increasing and convex in q. Assume also that C is increasing in θ. Assume also that $V(\cdot)$ is increasing and concave. Finally, we assume as much smoothness on C, V, and F_θ as we want. Consider a symmetric Baysian Nash equilibrium of the game.

Q1. (1pt) In this model, it turns out that the bidder's choice of quality does not depend on the strategies of the other bidders. Show that the bidder's equilibrium quality choice is the solution to the following problem:

$$\max_q V(q) - C(q, \theta_i). \tag{2}$$

Q2. (2pts) Define $q(\theta)$ to be the maximizer of (2). Define $K(\theta)$ to be the value of (2) at $q(\theta)$. Show that the equilibrium of the scoring auction game is equivalent to the equilibrium of the standard first-price sealed bid auction in which bidder type is now $K(\theta)$ and bids s.

Q3. (2pts) Use the observation in Q2 to show that the equilibrium distribution of $K(\theta)$ is identified from repeated observations of auctions (assume that you observe both p and q of every bidder). Explain how to estimate $K(\theta_i)$ and equilibrium costs, $C(q(\theta_i); \theta_i)$.

Q4. (1pt) Is F_θ identified? Explain.

Q5 (open ended, 1pt). Suppose your research goal is to evaluate the value of using scoring auctions relative to the standard price-only auctions. How would you go about getting an answer to that research question? Briefly discuss.

220C

Identify a research question related to Commitment.
(i) Motivate the research question; (ii) briefly talk about related papers; (iii) talk about what kind of data will help answer the question; and (iv) discuss briefly modeling and/or estimation strategies.