Field Exam: Advanced Theory

There are two questions on this exam, one for Econ 219A and another for Economics 206. Answer all parts for both questions.

Exercise 1: Consider a n-player all-pay auction auction with a single object where the highest bid wins and every agent i pays her bid t_i. Let θ_i be the value of agent i for the object, and assume that the values are i.i.d. distributed according to $F : [\underline{\theta}, \overline{\theta}] \to [0,1]$. The utility of the agent equals $\theta_i - t_i$ if she gets the object and $-t_i$ otherwise.

(a) Find a symmetric equilibrium of this auction. Is this an equilibrium in dominant strategies?

(b) What is the expected revenue in this auction.

(c) Is this all-pay auction revenue maximizing?

(d) Derive the bid in the corresponding first and second price auction.

(e) Compare the bid agent i makes in this auction to the bid she makes in a first or second price auction when she has the valuation θ_i. Explain intuitively, why the bids are ordered between the different auction formats.

(f) Now derive an equilibrium if there are $1 < k < n$ objects.

(g) Suppose now that the agent's utility is given by $\theta_i - c(t_i)$ if she gets the object and $-c(t_i)$ otherwise, where c is a strictly increasing function. Characterize a symmetric equilibrium bidding strategy.

Exercise 2: Consider a simple model of gym attendance (following DellaVigna and Malmendier 2004), where in period 0 individuals choose whether or not to sign a contract that requires them to pay a lump-sum membership fee L in period 1 and then an additional attendance price p if they attend the gym in period 2. They receive a health benefit b of attending the gym, which is a delayed benefit realized only later. They also incur a hassle cost c in period 2, which they feel immediately in period 2. In period 0 they only know that c will be drawn from a uniform distribution on the unit interval $[0,1]$, with c realized only at the beginning of period 2.

Individuals are present-biased, with a common present bias factor $\beta \leq 1$. They thus attend the gym in period 2 if and only if $\beta b - p - c \geq 0$. The present-biased individuals are sophisticated, and in period 0 they choose to sign the contract if $\beta \left[\int_{c=0}^{c=\beta b - p} (b-p-c)dc - L\right] \geq 0$.

Part a. Let $V(p,L)$ denote an individual's expected utility from signing the contract, from the period 0 perspective. Show that for $p < \beta b$,

$$V(p,L) = \beta \underbrace{(\beta b - p)}_{\text{P(attend)}} \underbrace{\left(b - \frac{\beta b + p}{2}\right)}_{\text{E(utility|attend)}} - \beta L$$

Part b. Show that $\frac{dV}{dL} = -\beta$ and $\frac{dV}{dp} = -\beta(b-p)$ for $p < \beta b$.

Part c. Suppose that the gym incurs a cost ψ whenever an individual attends the gym. And suppose also that p and L must satisfy the zero profit condition $L + Pr(\text{attend}) \cdot p = Pr(\text{attend}) \cdot \psi$. This zero profit condition allows us to write L as a function of p. What is $L(p)$?

Part d. If $L(p)$ is determined from the zero profit condition above, show that the value of p that maximizes $V(p, L(p))$ is given by $p^* = \psi - (1-\beta)b$.

Part e. Now let's generalized everything from parts a through f. If $L(p)$ is determined from the zero profit condition, prove that $p^* = \psi - (1-\beta)b$ without assuming that c is distributed uniformly on $[0,1]$; assume only that the distribution of c has a continuous density function with full support on the unit interval.

Part f. Please provide intuition for the p^* formula above. In particular, explain why $p^* = \psi$ when $\beta = 1$ and why $p^* < \psi$ when $\beta < 1$.

Part g. Suppose that the "gym economy" consists of many identical gyms, each of which incurs a cost ψ per attendance. Show that in a competitive equilibrium of this economy, gyms will set $p = \psi - (1-\beta)b$ and set L to satisfy the zero-profit condition.

Part h. Keep assuming the competitive equilibrium from part (g). Suppose that Calvin Voltt, a renowned researcher applying behavioral economics to health decisions, decides that it is a good idea to provide incentives for gym attendance to counteract the fact that most people seem to go to the gym less than they wanted to due to self-control problems. Calvin runs a large scale field experiment with a particular gym branch and finds that financial incentives do indeed increase gym attendance. Assuming that the gym branch maintains its standard pricing $p^* = \psi - (1-\beta)b$ during the experiment, explain why this field experiment actually created socially inefficient gym attendance while it was being run.

Part i. Keep assuming the competitive equilibrium from part (g). Suppose that Calvin cleverly measures people's present bias β and attendance health benefits b, and convinces the government to provide financial incentives of $r = (1-\beta)b$ per gym attendance. To maintain a balanced budget, these incentives must be funded through a lump-sum tax equal to $T = r \cdot Pr(\text{attend})$ per individual. Assume that the attendance incentives are obtained by individuals instantaneously in period 2, while the lump-sum tax is paid in period 1, alongside the membership fee L.

In the long-run, the competitive gym economy will adjust its contract terms (L, p) in response to this government incentive policy. Show that when the equilibrium adjusts, gyms will set $p = \psi$, and the net effect of the government intervention on individuals' (long-run) welfare will be zero.

Part j. Please comment on the broad lesson that parts (h) and (i) are conveying about "behaviorally-informed" interventions.

Part k. How would things change if consumers were (partially) naive and over-estimated their future self-control? In particular, would the interventions in parts (h) and (i) be more helpful, less helpful, or equally helpful in this case? Feel free to solve for the equilibrium p^* as a function of the actual β as well as what consumers in period 0 think their β will be in period 2, denoted $\hat{\beta}$.