Theory Field Examination
Behavioral economics (219A)
Aug. 2013

Problem 1. Suppose that Jay ("J") will live for $T = 3$ periods (periods 1, 2, and 3). In each of these three periods he chooses whether to consume sugary drinks, $a_t = 1$, or not to consume sugary drinks, $a_t = 0$. Sugary drinks are addictive, and J (who can get such drinks for free and has no other source of pleasure or displeasure in life) has preferences in each of periods 1, 2, and 3 of the form:

$$\begin{aligned} u_t(a_t = 0 | a_{t-1} = 0) &= 0 \\ u_t(a_t = 1 | a_{t-1} = 0) &= 1 \\ u_t(a_t = 0 | a_{t-1} = 1) &= -\infty \\ u_t(a_t = 1 | a_{t-1} = 1) &= -1 \end{aligned}$$

These preferences capture the notion that consuming in the previous period makes J addicted: it means he gets less utility from continuing to consume sugary drinks than he does from consuming for the first time – he no longer gets the burst of pleasure and even gets a "hangover" – , but withdrawing is very unpleasant. J dies at the beginning of Period 4, and his utility from that point on doesn't depend on what he does in his lifetime. Importantly, assume that $a_0 = 0$ in all parts below: J is born unaddicted.

Suppose that J has (quasi-)hyperbolic discounting preferences with $\beta < 1$ and $\delta = 1$. J might be either sophisticated or naive. For all questions below, don't worry about specifying behavior for any knife-edge values of β that makes J indifferent among choices in any contingency.

a) What will J do in Period 3, as a function of β, whether he is sophisticated or naive, and whether he enters the period having chosen $a_2 = 0$ or $a_2 = 1$ in the previous period? Briefly give an intuition for you answer.

b) As a function of β, what pattern of drinking (a_1, a_2, a_3) will we observe J choosing if he is naive?

c) As a function of β what pattern of drinking (a_1, a_2, a_3) will we observe J choosing if he is sophisticated?

d) Briefly interpret and give an intuition for any similarities or differences in your answers to parts (b) and (c).

Now suppose that $\beta = 1$. But now J may suffer from projection bias in predicting his future preferences. In any period in which he drank last period ($a_{t-1} = 1$) J predicts his future utilities in all future periods are given by:

$$
\begin{aligned}
u_t(a_t = 0|a_{t-1} = 0) &= \alpha(-\infty) + (1-\alpha)0 = -\infty \\
u_t(a_t = 1|a_{t-1} = 0) &= \alpha(-1) + (1-\alpha)(1) = 1 - 2\alpha \\
u_t(a_t = 0|a_{t-1} = 1) &= -\infty \\
u_t(a_t = 1|a_{t-1} = 1) &= -1
\end{aligned}
$$

and in any period (including period 1) in which he did not drink last period ($a_{t-1} = 0$) J predicts his future utilities in all future periods are given by:

$$
\begin{aligned}
u_t(a_t = 0|a_{t-1} = 0) &= 0 \\
u_t(a_t = 1|a_{t-1} = 0) &= 1 \\
u_t(a_t = 0|a_{t-1} = 1) &= \alpha(0) + (1-\alpha)(-\infty) = -\infty \\
u_t(a_t = 1|a_{t-1} = 1) &= \alpha(1) + (1-\alpha)(-1) = 2\alpha - 1
\end{aligned}
$$

where $\alpha \in [0, 1)$.

e) As a function of α, what pattern of drinking (a_1, a_2, a_3) will we observe J choosing? Don't worry about specifying behavior for knife-edge values of α where J might be indifferent in some contingencies. Briefly give an intuition for your answer.

Now suppose the mayor of the town J resides in, let's call him Mayor BB, has gotten wind from public health researchers that people may be overconsuming sugary drinks (even abstracting from negative downstream health consequences, like increased risk of diabetes) and is considering a ban. J is a representative agent of Mayor BB's town and the public health researchers have presented the mayor with perfect estimates of J's hedonic or experienced utility function, $u_t(\cdot)$ ($\forall\, t$).

f) What pattern of drinking (a_1, a_2, a_3) do the public health researchers inform the mayor is optimal, i.e., what pattern (a_1^*, a_2^*, a_3^*) maximizes $u_1 + u_2 + u_3$? Briefly compare this optimal pattern of drinking with the pattern you found in part (b), where you assumed J to be a naive hyperbolic discounter.

g) Continue to suppose J is a naive hyperbolic discounter. As a function of *J's pattern of drinking* (a_1, a_2, a_3) – importantly, *not* as a function of β – compare his experienced utility with his utility if sugary drinks were banned, in which case J would have to consume $a_1 = a_2 = a_3 = 0$. For which observed pattern of behavior absent the ban (a_1, a_2, a_3) does the Mayor calculate that a ban is optimal (with the help of the public health researchers' estimates of hedonic utility)?

h) Would the answer to part (g) change if J is a sophisticated hyperbolic discounter? How about if he has projection bias?

i) Does your answer to part (h) suggest a response to the following question: Supposing Mayor BB knows J's pattern of drinking absent a ban (as well as the public health researchers' estimates of hedonic utility) then would he find it helpful to additionally know the details of J's underlying psychology that drive this pattern – e.g., whether he has projection bias or is a naive or sophisticated hyperbolic discounter – in order to calculate whether a ban is optimal? Please explain. (In your answer, feel free to discuss whether you think this question is misleading in some way.)

Theory Field Examination
Game Theory (209A)
Aug. 2013

Good luck!

Question 1 (strategic games)

- Consider the variant of the Hawk-Dove game

	D	H
D	$1,1$	$0,2$
H	$2,0$	$1-c,1-c$

  (when $c > 1$ the game has the standard Hawk-Dove structure). Find the set of *all* Nash equilibria for all values of c. Are the equilibrium strategies evolutionary stable?

- Give an example of a game with a *unique* mixed strategy Nash equilibrium in which each player equilibrium payoff exceeds her max min payoff.

- Let α^* be an evolutionary stable strategy. Does α^* necessarily weakly dominates every other strategy? Is it possible that some strategy weakly dominates α^*? Does it matter if α^* is pure or mixed?

Question 2 (repeated games)

Consider the general Prisoner's Dilemma game

	C	D
C	x,x	$0,y$
D	$y,0$	$1,1$

where $y > x > 1$.

- Find the condition on x, y, and the discount factor $0 < \delta < 1$ such that a pair of grim trigger strategies is a Nash equilibrium for the infinitely repeated game.

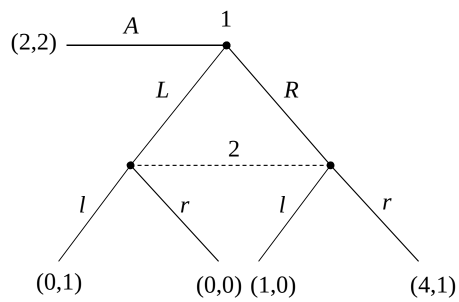
- Find conditions on k, x, y, and the discount factor δ such that a pair of k-period limited punishment strategies is a Nash equilibrium for the infinitely repeated game.

– Find the approximate set of discounted average equilibrium payoffs (the set of enforceable payoffs).

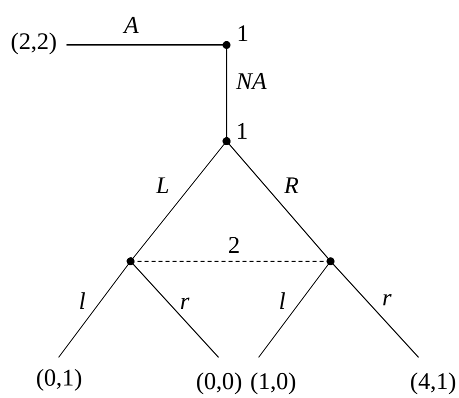
Question 3 (extensive games of imperfect information)

Find the sets of sequential equilibria of the two games in the figures attached below (Game II is obtained from Game I by adding a move to player 1). Discuss the differences between the sets of equilibria in the two games.

Game I



Game II



THEORY FIELD EXAM: AGENCY & MECHANISM DESIGN
AUGUST 2013

Consider a standard hidden-action principal-agent game with the following timing and assumptions. A principal makes an agent a take-it-or-leave-it (TIOLI) offer. If the agent leaves it, he gets utility 0. If he accepts, he expends effort $a \in \mathcal{A}$ and receives a payment contingent on an outcome, $x \in \mathcal{X}$. Assume the agent's utility is $u(y) - c(a)$, where y is a payment from principal to agent; $u : (\underline{y}, \infty) \to \mathbb{R}$ is an increasing and concave function; and $c : \mathcal{A} \to \mathbb{R}_+$. Assume $\lim_{y \to \underline{y}} u(y) = -\infty$ and $\lim_{y \to \infty} u(y) = \infty$.

(a) Suppose that $\mathcal{X} = \{x_0, x_1\}$, $\mathcal{A} = \{1, 2\}$, $\Pr\{x = x_1 | a\} = 1 - 1/a$, and $c(1) < c(2)$. What contract would the principal offer in equilibrium if she wishes to induce $a = 2$?

(b) Same assumptions as in part (a), except assume $\mathcal{A} = [1, 2]$ and $c(a) = a$. What contract would the principal offer in equilibrium if she wishes to induce $\hat{a}$, $\hat{a} \in (1, 2]$?

Now consider the following variation. Let $\mathcal{A} = [0, 1]$. Suppose that a state of nature, θ, is drawn uniformly from the interval $[0, 1]$. This occurs prior to the principal's offering the agent a contract on a TIOLI basis. Assume that $\mathcal{X} = \{x_0, x_1\}$ and $\Pr\{x = x_1 | a\} = \beta a$, $\beta \in (0, 1)$. Suppose that

$$c(a) = \begin{cases} 0\,, & \text{if } a \leq \theta \\ (a - \theta)^2\,, & \text{if } a > \theta \end{cases} .$$

(c) Suppose neither principal nor agent know the realized value of θ at the time of contracting. Suppose, too, that the agent is also ignorant of the realized value of θ when he chooses his action (he only infers θ after choosing his action when he realizes its cost). The principal never observes θ nor can the agent demonstrate its value to the principal. Suppose the principal wishes to induce $\hat{a} \in (0, 1]$. What contract would she offer in equilibrium?

Maintain the assumptions made so far, except consider the following variant of the problem. Suppose that $u(y) = y$, $\underline{y} = -\infty$, $x_n = n$ ($n \in \{0, 1\}$), and that the agent learns θ prior to contracting with the principal. This is the agent's private (hidden) information. Assume, now, that θ is drawn uniformly from $[0, 1/2]$. Finally, assume the principal's utility is $x - y$.

(d) What contract would the principal offer in equilibrium? [Hint: note, now, the principal may wish for the action to depend on the agent's type (*i.e.*, θ).]