Theory Field Examination

January 2021

Question A (Economics 207a)

1. Construct a set of priors $C \in \Delta S$ on a state space S such that the utility function $U : [0,1]^S \to \mathbb{R}$ defined by

$$U(f) = \min_{p \in C} \int_S f \, dp$$

does not satisfy comonotonic independence (in the sense of Schmeidler). Recall $\succsim$ satisfies comonotonic independence if $f \succsim g$ if and only if $\alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h$, for all $\alpha \in (0,1)$ and f, g, h are pairwise comonotonic (that is, if every pair chosen among the three acts is comonotonic). In this application, f and g are comonotonic if $f(s) \geq f(t) \iff g(s) \geq g(t)$.

2. Let X be a finite set of consequences and S be a finite set of states. Fix some set of priors $C \subset \Delta S$ and an affine (expected-utility) function $v : X \to \mathbb{R}$. Consider the binary relation $\succsim$ on $(\Delta X)^S$ defined by $f \succsim g$ if and only if there exists some $p \in C$ such that

$$\int_S v \circ f \, dp \geq \int_S v \circ f \, dp.$$

Observe that $\succsim$ is generally intransitive. Prove or provide a counterexample to the following statements:

   (a) $\succsim$ has convex upper contour sets.

   (b) $\succsim$ satisfies independence, in the sense that $f \succsim g$ if and only if $\alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h$.

3. Gul and Pesendorfer (2001) say the following defines a **overwhelming temptation** representaton:

$$U(A) = \max_{x \in A} u(x) \text{ subject to } v(x) \geq v(y) \text{ for all } y \in A$$

Suppose $\succsim$ admits an overwhelming temptation representation. Prove or provide a counterexample to the following claim:

   (a) $\succsim$ satisfies Set Betweenness. (That is, $A \succsim B$ implies $A \succsim A \cup B \succsim B$.)

   (b) $\succsim$ satisfies Independence. (That is, $A \succsim B$ if and only if $\alpha A + (1-\alpha)C \succsim \alpha B + (1-\alpha)C$, for all $\alpha \in (0,1)$ and A, B, C are convex sets of lotteries.)

Problem for Econ 207B

1. Consider the following indivisible object assignment model. Let N be a finite set of agents, and X be a finite set of indivisible objects such that $|N| \leq |X|$. Let $\mathcal{R}$ denote the set of linear orders over X. Let $\mathcal{A}$ denote the set of all one-to-one functions $\mu : N \to X$. A mechanism is a function $f : \mathcal{R}^N \to \mathcal{A}$. For all $i \in N$ and $R \in \mathcal{R}^N$, let $f_i(R) = \mu(i)$ if $\mu = f(R)$. For all $R \in \mathcal{R}^N$ and $M \subset N$, let $R_M = (R_i)_{i \in M}$.

   (a) When $|N| = 2$, characterize the set of all strategy-proof and Pareto efficient mechanisms.

   (b) In this part, let $|N|$ be arbitrary. A mechanism f is **group strategy-proof** if for all nonempty $M \subset N$, $R \in \mathcal{R}^N$, and $R'_M \in \mathcal{R}^M$:

   $$\big[\forall i \in M\colon f_i(R'_M, R_{N\setminus M}) R_i f_i(R)\big] \implies \big[\forall i \in M\colon f_i(R'_M, R_{N\setminus M}) = f_i(R)\big]$$

   A mechanism f is **non-bossy** if for all $i \in N$, $R \in \mathcal{R}^N$, and $R'_i \in \mathcal{R}$:

   $$f_i(R'_i, R_{N\setminus\{i\}}) = f_i(R) \implies \big[\forall j \in N\colon f_j(R'_i, R_{N\setminus\{i\}}) = f_j(R)\big]$$

   Prove that a mechanism is group strategy-proof if and only if it is non-bossy and strategy-proof.